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RESUME OF THE THEORY OF PLANE SHOCK AND ADIABATIC WAVES  
WITH APPLICATIONS TO THE THEORY OF THE SHOCK TUBE

C. W. Lampson

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ABERDEEN PROVING GROUND, MARYLAND

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WITH APPLICATIONS TO THE THEORY OF THE SHOCK TUBE**

A reprint and extension of a Technical Memorandum written for the Princeton University Station, Division 2, NDRC, 27 April 1945 by C. W. Lampson, now Chief, Ordnance Engineering Laboratory, Ballistic Research Laboratories.

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**ABERDEEN PROVING GROUND, MARYLAND**

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CWLampson/lbe  
Aberdeen Proving Ground, Md.  
27 March 1950

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APPLICATIONS TO THE THEORY OF THE SHOCK TUBE

ABSTRACT

The theory of plane shock and adiabatic waves is presented in an easily derived manner together with sufficient background material to enable the novice in the field to grasp the fundamentals required for further study. The application of the basic theory to the shock tube as a research instrument is given together with some experimental results to illustrate the calculations. Certain conceptions of energy and its relation to the impulse in a shock wave are presented in a manner not used in the literature.

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# TABLE OF SYMBOLS

E	internal energy in a unit mass of gas
I	impulse in a shock wave
M	Mach number or ratio of particle velocity to local sound velocity behind the shock front
P	absolute pressure
$P_0$	pressure in region into which shock wave advances (usually atmospheric)
$P_f$	excess pressure in reflected shock wave
$P_s$	excess pressure in shock front
U	velocity of shock front
a	local velocity of sound
$a_0$	velocity of sound at pressure $P_0$
c	velocity of sound behind shock front
K	constant
m	mass of gas
t	time
u	particle velocity behind the shock front
$v$	velocity of propagation of a section of a shock wave
$y = \frac{P}{P_0}$	
$z = \frac{P - P_0}{P_0} = \frac{P_s}{P_0} = y - 1$	
$\gamma = \frac{C_p}{C_v}$	or ratio of specific heats (for air = 1.4)
$\omega = \frac{P_0}{P}$	
$\rho$	density of the gas

subscript o refers to conditions of the medium into which the shock wave advances

subscripts 1, 2, 3, etc. refer to conditions in various sections of the shock wave

subscript c refers to conditions in the compression chamber of the shock tube

subscript r refers to conditions in a reflected shock wave

With Reference to Section VII

- $b_o$  velocity of initial tip of rarefaction wave back into compression chamber
- $b_1$  velocity of initial tip of rarefaction wave in region of cool gas
- $b_2$  velocity of initial tip of rarefaction wave in region of hot gas
- $V_o$  velocity of trough of rarefaction wave back into compression chamber
- $V_1$  velocity of trough of rarefaction wave in region of cool gas
- $V_2$  velocity of trough of rarefaction wave in region of hot gas
- $L_o$  length of compression chamber of shock tube
- $L_e/L_o$  ratio of length of expansion chamber to length of compression chamber
- d distance that hot gas boundary moves in time T

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## INTRODUCTION

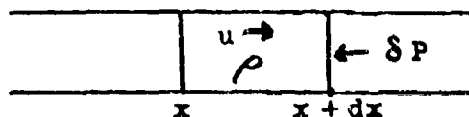
The theory of shock waves has been covered in many excellent papers by British and American authors, notably G. I. Taylor and W. G. Penney of England, and J. von Neumann, J. Kirkwood, G. Kistiakowsky and E. B. Wilson, S. Chandrasekhar, and others of this country. It is the purpose of this paper to present some of their results in an easily derived manner together with certain background material which would make it possible for the uninitiated quickly to gain a certain perspective in the field with the object of applying this background to applications of the shock tube as a research instrument. Certain conceptions of energy and its relation to the impulse in a shock wave are presented in a manner not used in the literature, and experimental results are quoted to show the validity of certain assumptions necessary for such relationships to hold.

The use of a bursting diaphragm in a tube as a method of producing shock waves is covered in a phenomenological report by W. Payman and W. C. F. Shepherd in 1941. They attribute its earliest conception to P. Vielle in 1899. A calculation of the shock wave pressures to be expected thereby was carried out by A. H. Taub in 1942. Certain experimental work on the tube as a primary standard of blast wave pressures was done by G. T. Reynolds, and extended in a series of quite accurate measurements by W. T. Read who found that the experimental pressures varied about 6 percent from those calculated from tube theory. L. G. Smith has used the tube as an aid in studying experimentally the reflection of shock waves at oblique incidence. It has been used as an instrument in the study of bursting diaphragms from incident shock waves and as an adjunct to the development of piezo-electric pressure gauges. Certain proposals have been made for using the tube on a fairly large scale for tests on land mines and other devices exposed to explosive blast in an effort to reproduce field results in the laboratory.

It is the earnest hope that this paper may provide sufficient background so that persons who have not previously used the tube may readily understand in a quantitative way the phenomena that may be observed.

### I. THE PROPAGATION OF FINITE ADIABATIC WAVES IN A TUBE

The derivation of the properties of one-dimensional finite waves can be carried out quite readily by considering a slice of gas in a tube bounded by planes at  $x$  and at  $x + dx$  as shown below.



The gas in the thin slice is assumed to have a density  $\rho$  and a particle velocity  $u$ , both of which are functions of time. Then one can

write for the forces on the slice

$$\rho \delta x \frac{d}{dt}(u) = - \delta P \quad \text{where } u = f(t) \quad (I.1)$$

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} \frac{dx}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \quad \text{and} \quad \frac{\delta P}{\delta x} = \frac{\partial P}{\partial x}$$

So from equation (I.1) we have the familiar equation of motion for a gas in a tube.

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} = 0 \quad (I.2)$$

The equation of continuity is

$$\frac{\partial}{\partial x}(\rho u) = - \frac{\partial \rho}{\partial t} \quad (I.3)$$

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \quad \text{but since } \rho = f(u)$$

$$\frac{\partial \rho}{\partial t} = \frac{d\rho}{du} \frac{\partial u}{\partial t} \quad \text{and} \quad \frac{\partial \rho}{\partial x} = \frac{d\rho}{du} \frac{\partial u}{\partial x}$$

we have, after making indicated changes of variable,

$$\frac{d\rho}{du} \frac{\partial u}{\partial t} + u \frac{d\rho}{du} \frac{\partial u}{\partial x} + \rho \frac{du}{d\rho} \frac{d\rho}{du} \frac{\partial u}{\partial x} = 0$$

Cancelling  $\frac{d\rho}{du}$  the equation of continuity may be written as

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \rho \frac{du}{d\rho} \frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + (u + \rho \frac{du}{d\rho}) \frac{\partial u}{\partial x} = 0 \quad (I.3a)$$

Since  $P = f(\rho)$  and  $\rho = f(u)$  we have that

$$\frac{\partial P}{\partial x} = \frac{dP}{d\rho} \frac{\partial \rho}{\partial x} \quad \text{and} \quad \frac{\partial \rho}{\partial x} = \frac{d\rho}{du} \frac{\partial u}{\partial x}$$

so that

$$\frac{\partial P}{\partial x} = \frac{dP}{d\rho} \frac{d\rho}{du} \frac{\partial u}{\partial x}$$

Substituting this expression into the equation of motion (I.2), we see that it may be written as

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{dP}{d\rho} \frac{d\rho}{du} \frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + (u + \frac{1}{\rho} \frac{dP}{d\rho} \frac{d\rho}{du}) \frac{\partial u}{\partial x} = 0 \quad (I.2a)$$

These two equations (I.2a and I.3a) may be made consistent if

$$\rho \frac{du}{d\rho} = \frac{1}{\rho} \frac{dP}{d\rho} \frac{d\rho}{du} \quad \text{or} \quad \left(\frac{du}{d\rho}\right)^2 = \frac{1}{\rho^2} \frac{dP}{d\rho} \quad (I.4)$$

then 
$$\frac{du}{d\rho} = \pm \frac{1}{\rho} \sqrt{\frac{dP}{d\rho}} \quad (\text{I.4a})$$

and therefore 
$$u = \pm \int_{\rho_0}^{\rho} \sqrt{\frac{dP}{d\rho}} \frac{d\rho}{\rho} \quad (\text{I.5})$$

This is the expression for the particle velocity in terms of the pressure and density for waves which travel in one direction.

Returning to the equation (I.2a) of motion we see that

$$\frac{d\rho}{du} = \pm \rho \sqrt{\frac{d\rho}{dP}} \quad \text{from equation (I.4a)}$$

so that we have

$$\begin{aligned} \frac{\partial u}{\partial t} + (u \pm \frac{dP}{d\rho} \sqrt{\frac{d\rho}{dP}}) \frac{\partial u}{\partial x} &= 0 \text{ which equals} \\ \frac{\partial u}{\partial t} + (u \pm \sqrt{\frac{dP}{d\rho}}) \frac{\partial u}{\partial x} &= 0 \end{aligned} \quad (\text{I.6})$$

For an adiabatic compression or expansion  $\frac{P}{\rho^\gamma} = k$

$$\frac{dP}{d\rho} = \gamma k \rho^{\gamma-1} = \gamma \frac{P}{\rho} \rho^{\gamma-1} = \frac{\gamma P}{\rho}$$

but  $\frac{\gamma P}{\rho} = a^2$  where  $a$  is the local velocity of sound in the medium then

$$\sqrt{\frac{dP}{d\rho}} = a$$

and the equation of motion reduces to

$$\frac{\partial u}{\partial t} + (u \pm a) \frac{\partial u}{\partial x} = 0 \quad (\text{I.6a})$$

If we wish to find the velocity of a section of the wave of constant particle velocity and consequently constant pressure we can do so by letting  $u$  be a constant so that  $du = 0$

but since  $u = f(x, t)$  
$$du = \frac{\partial u}{\partial t} dt + \frac{\partial u}{\partial x} dx = 0$$

so that 
$$\frac{dx}{dt} = - \frac{\frac{\partial u}{\partial t}}{\frac{\partial u}{\partial x}} \quad \text{velocity of propagation of that section of the wave}$$

But from (I.6a) we see that

$$- \frac{\frac{\partial u}{\partial t}}{\frac{\partial u}{\partial x}} = (u \pm a) = \frac{dx}{dt}$$

so that the velocity of propagation of a section of the wave of constant particle velocity is equal to  $(a \pm u)$ .

$$v = a \pm u \quad (I.7)$$

The sign of  $u$  is positive if the particle velocity has the same direction as the wave propagated in the medium and is negative if the two are in opposite directions. The local velocity of sound  $a$  will be a function of the pressure  $P$  as will the particle velocity  $u$ .

If the wave is an adiabatic compression (not a shock wave) then we may evaluate the particle velocity  $u$  and the local velocity of sound as follows: Assume that  $P = P_0 \left(\frac{\rho}{\rho_0}\right)^\gamma$  for an adiabatic pressure change

$$\begin{aligned} \text{then } \frac{dP}{d\rho} &= \gamma P_0 \left(\frac{\rho}{\rho_0}\right)^{\gamma-1} \\ \therefore u &= \pm \int_{\rho_0}^{\rho} \frac{\gamma P_0 \left(\frac{\rho}{\rho_0}\right)^{\gamma-1}}{\rho_0^{\frac{1}{2}}} \cdot \frac{d\rho}{\rho} = \pm \int_{\rho_0}^{\rho} \left(\frac{\gamma P_0}{\rho_0}\right)^{\frac{1}{2}} \rho^{\frac{\gamma-3}{2}} d\rho \\ &= \pm \left[ \left(\frac{\gamma P_0}{\rho_0}\right)^{\frac{1}{2}} \cdot \frac{1}{\rho_0^{\frac{\gamma-1}{2}}} \cdot \frac{2}{\gamma-1} \left(\rho^{\frac{\gamma-1}{2}} - \rho_0^{\frac{\gamma-1}{2}}\right) \right] \\ &= \pm \frac{2}{\gamma-1} \left(\frac{\gamma P_0}{\rho_0}\right)^{\frac{1}{2}} \left[\left(\frac{\rho}{\rho_0}\right)^{\frac{\gamma-1}{2}} - 1\right] \end{aligned} \quad (I.8)$$

but  $\left(\frac{\gamma P_0}{\rho_0}\right)^{\frac{1}{2}} = a_0$  the velocity of sound in the medium into which the wave advances.

$$\text{So that} \quad u = \pm \frac{2}{\gamma-1} a_0 \left[\left(\frac{\rho}{\rho_0}\right)^{\frac{\gamma-1}{2}} - 1\right] \quad (I.8a)$$

$$\begin{aligned} \text{but } \frac{\rho}{\rho_0} &= \left(\frac{P}{P_0}\right)^{\frac{1}{\gamma}} \\ \text{consequently} \quad u &= \pm \frac{2}{\gamma-1} a_0 \left[\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{2\gamma}} - 1\right] \end{aligned} \quad (I.8b)$$

particle velocity for a compressional wave.

In the same manner we find that the particle velocity for a rarefaction wave is

$$u = \pm \frac{2}{\gamma-1} a_0 \left[1 - \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{2\gamma}}\right] \quad (I.9)$$

Where  $a_0$  is again the velocity of sound in the medium into which the wave advances.

The local velocity of sound  $a$  is related to the velocity of sound  $a_0$  in a medium of pressure  $P_0$  and density  $\rho_0$  as follows:

$$a^2 = \frac{\gamma P}{\rho} = \frac{\gamma P_0}{\rho_0} \frac{P}{P_0} \frac{\rho_0}{\rho} = \frac{\gamma P_0}{\rho_0} \cdot \left(\frac{P}{P_0}\right) \left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}} = \frac{\gamma P_0}{\rho_0} \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

so that  $a = a_0 \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{2\gamma}}$  (I.10)

The velocity of a section of compressional wave of constant particle velocity advancing into an undisturbed medium is

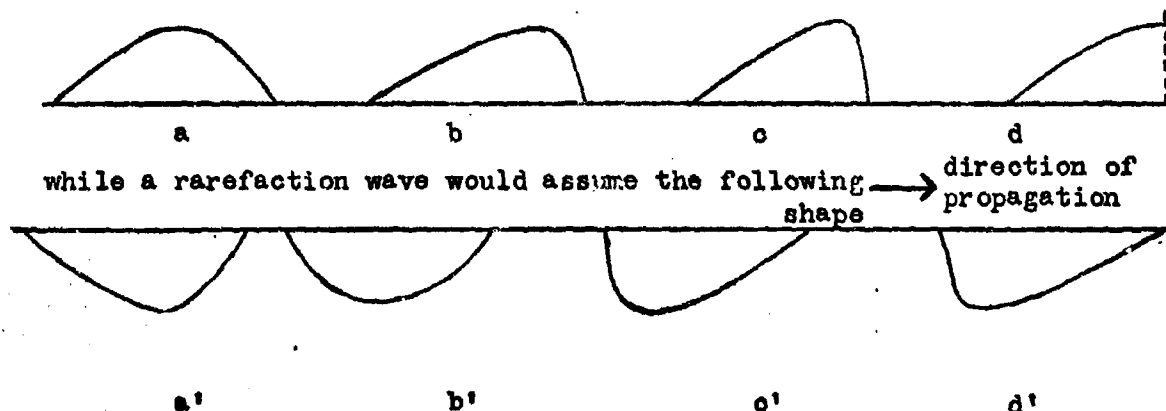
$$\begin{aligned} v_{u=\text{constant}} &= u + a = a_0 \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{2\gamma}} + \frac{2}{\gamma-1} a_0 \left[ \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{2\gamma}} - 1 \right] \\ &= a_0 \left[ \frac{\gamma+1}{\gamma-1} \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{2\gamma}} - \frac{2}{\gamma-1} \right] \end{aligned}$$

If  $\gamma = 1.4$  then

$$v_{u=\text{constant}} = a_0 \left[ 6 \left(\frac{P}{P_0}\right)^{1/7} - 5 \right] \quad (\text{I.10a})$$

This is the velocity also for a rarefaction wave advancing into a medium of pressure  $P_0$  and density  $\rho_0$  and particle velocity  $u_0 = 0$ .

From equation (I.10a) it can be seen that the higher pressure parts of a wave will travel faster than the lower pressure parts so that a finite compressional wave would assume the shapes shown below in time sequence.



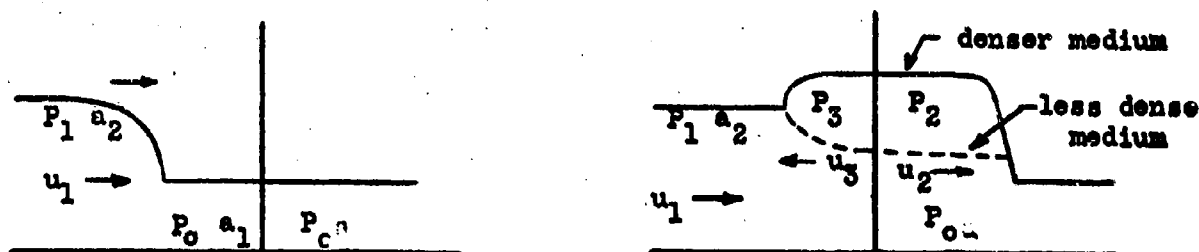
A compressional wave tends to assume a steeper slope at the front of the wave while a rarefaction wave tends to become less steep and to change shape in the opposite direction. The case (d) of the compressional wave can not happen for the front has a double value of pressure but the front does become vertical with the formation of a shock wave which does not obey the same equations as does the adiabatic waves.

The rarefaction waves however obey the adiabatic equations at least to the trough of the wave. It can be seen that there is a tendency to form a secondary shock in the tail of the rarefaction wave. This phenomenon will not be treated here. The shock wave when formed obeys a set of conditions embodied in the Rankine-Hugoniot equation which will be derived in a succeeding section.

## II. THE REFLECTION AND TRANSMISSION OF ADIABATIC WAVES AT A DISCONTINUITY

### (a) Compressional Waves.

The sequence of events where an adiabatic wave meets a discontinuity can be shown by the following sketch.



The velocity of sound on the right of the boundary is  $a$  and on the left of the boundary is  $a_1$ . The pressure initially is the same on both sides and is  $P_0$ . An adiabatic wave moving to the right with a pressure  $P_1$ , particle velocity  $u_1$ , and velocity of sound behind front of  $a_2$ , approaches the boundary. Depending on the conditions a wave of greater or lesser amplitude will be transmitted, while a compressional or rarefaction wave will be reflected to the left. The pressure and the particle velocity behind the transmitted wave are  $P_2$  and  $u_2$ , respectively, while the pressure and the particle velocity of the reflected wave are  $P_3$  and  $u_3$ .

The two conditions which must be satisfied at the boundary are

$$P_2 = P_3$$

pressures equal (II.1)

and  $u_1 - u_3 = u_2$

particle velocities equal (II.2)

The values of the particle velocities are (if  $\gamma = 1.4$ )

$$u_1 = 5a_1 \left[ \left( \frac{P_1}{P_0} \right)^{1/7} - 1 \right] \quad (\text{II.3})$$

$$u_2 = 5a \left[ \left( \frac{P_2}{P_0} \right)^{1/7} - 1 \right] \quad (\text{II.4})$$

$$u_3 = 5a_2 \left[ \left( \frac{P_3}{P_1} \right)^{1/7} - 1 \right] \quad (\text{II.5})$$

but  $a_2 = a_1 \left( \frac{P_1}{P_0} \right)^{1/7}$

so  $u_3 = 5a_1 \left[ \left( \frac{P_3}{P_0} \right)^{1/7} - \left( \frac{P_1}{P_0} \right)^{1/7} \right] \quad (\text{II.5a})$

Then from equation (II.2) we have

$$5a_1 \left[ \left( \frac{P_1}{P_0} \right)^{1/7} - 1 - \left( \frac{P_3}{P_0} \right)^{1/7} + \left( \frac{P_1}{P_0} \right)^{1/7} \right] = 5a_2 \left[ \left( \frac{P_2}{P_0} \right)^{1/7} - 1 \right] \quad (\text{II.6})$$

and since  $P_2 = P_3$  from equation (II.1) we have

$$\left( \frac{P_2}{P_0} \right)^{1/7} = \frac{2a_1}{a + a_1} \left( \frac{P_1}{P_0} \right)^{1/7} + \frac{a - a_1}{a + a_1} \quad (\text{II.7})$$

or in more useful form

$$P_2^{1/7} = \frac{2a_1}{a + a_1} P_1^{1/7} + \frac{a - a_1}{a + a_2} P_0^{1/7} \quad (\text{II.8})$$

If the boundary is a rigid wall so that  $u_2 = 0$  then we have

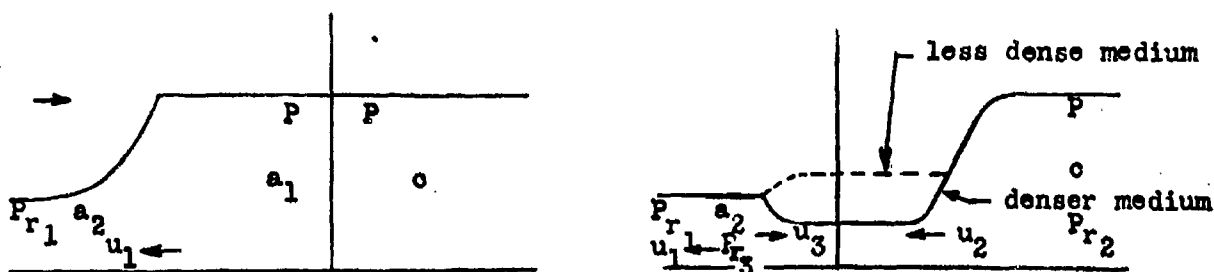
$$u_1 = u_3 \quad (\text{II.9})$$

and  $a_1 \left[ \left( \frac{P_1}{P_0} \right)^{1/7} - 1 \right] = a_1 \left[ \left( \frac{P_3}{P_0} \right)^{1/7} - \left( \frac{P_1}{P_0} \right)^{1/7} \right] \quad (\text{II.10})$

or  $P_3^{1/7} = 2P_1^{1/7} - P_0^{1/7} \quad (\text{II.11})$

(b) Rarefaction Waves.

We show a similar sketch for the sequence of events with a slightly different notation to adapt it to the notation in the other sections,



Where  $P$  and  $c$  are the pressure and sound velocity on the right of the boundary and  $P$  and  $a_1$  are the pressures and sound velocities on the left of the boundary. A rarefaction wave of pressure  $Pr_1$ , particle velocity  $u_1$ , and sound velocity behind the front of  $a_2$ , approaches from the left. Depending on the values of  $a_1$  and  $c$  a rarefaction or a small compression wave is reflected, while a rarefaction of greater or lesser amplitude is transmitted. The pressure and particle velocity behind the transmitted wave is  $Pr_2$  and  $u_2$  while those behind the reflected wave are  $Pr_3$  and  $u_3$  respectively.

The conditions to be satisfied are

$$Pr_2 = Pr_3 \quad (II.1a)$$

and  $u_1 - u_3 = u_2 \quad (II.2a)$

where  $u_1 = 5a_1 \left[ 1 - \left( \frac{Pr_1}{P} \right)^{1/7} \right] \quad (II.12)$

$$u_2 = 5c \left[ 1 - \left( \frac{Pr_2}{P} \right)^{1/7} \right] \quad (II.13)$$

$$u_3 = 5a_2 \left[ 1 - \left( \frac{Pr_3}{Pr_1} \right)^{1/7} \right] \quad (II.14)$$

but  $a_2 = a_1 \left( \frac{Pr_1}{P} \right)^{1/7}$

so  $u_3 = 5a_1 \left[ \left( \frac{Pr_1}{P} \right)^{1/7} - \left( \frac{Pr_3}{P} \right)^{1/7} \right] \quad (II.14a)$

Equating the particle velocities we have since  $Pr_2 = Pr_3$

$$(Pr_2)^{1/7} = \frac{2a_1}{c + a_1} Pr_1^{1/7} + \frac{c - a_1}{c + a_1} P^{1/7} \quad (II.15)$$

Similarly for reflection against a rigid wall where  $u_2 = 0$  we

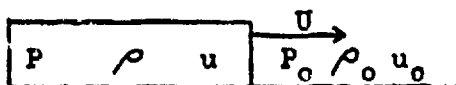
have the conditions

$$\frac{Pr}{3}^{1/7} = 2 \frac{Pr}{1}^{1/7} - p^{1/7} \quad (\text{II.16})$$

These are seen to be identical in form with those derived for compressional waves.

### III. DERIVATION OF THE RANKINE-HUGONIOT EQUATIONS FOR A SHOCK WAVE

If a shock wave travels with a velocity  $U$  into undisturbed air (air at velocity  $u_0 = 0$  in which the pressure is  $P_0$  and the density is  $\rho_0$ ) and if the air behind the shock is at a pressure  $P$ , density  $\rho$ , and is moving with a velocity  $u$ , then, by using the fact that for a unit mass of air crossing the shock front we must have mass, momentum and energy conservation, we obtain equations involving  $U$ ,  $P$ ,  $\rho$ ,  $u$ ,  $P_0$ ,  $\rho_0$  and  $u_0$  which are known as the Rankine-Hugoniot equations. We derive them as follows:



Let us consider an observer that moves with the shock front. In one second the amount of matter that crossed (from the right) a unit cross section of the wave front is  $\rho_0 U$ . This must equal the amount that gets away from the left face of the cross section in the same interval of time, viz.,  $\rho (U - u)$ . Hence we obtain the conservation of mass equation.

$$\rho (U - u) = \rho_0 U = m \quad \text{conservation of mass (III.1)}$$

The momentum of the mass  $\rho_0 U$  is  $\rho_0 U^2$  and the momentum of the mass  $\rho (U - u)$  is  $\rho (U - u)^2$ . The change of momentum across the shock front must equal the force acting. This is the difference of pressure on the two sides of the front times the cross section which we have taken as unity. Hence we obtain the conservation of momentum equation

$$P - P_0 = \rho_0 U^2 - \rho (U - u)^2$$

or

$$P + \rho (U - u)^2 = P_0 + \rho_0 U^2 \quad \text{conservation of (III.2) momentum.}$$

To obtain the energy equation we need to know the internal energy of the unit mass of gas (when it is at a pressure  $P_0$  and density  $\rho_0$ ). This is the work done against external pressure when the gas is expanded adiabatically to zero density. We call these internal energies  $E$  and  $E_0$ .

The work done by pressure per unit area per second on a column of gas of unit cross section (the column extends through the shock front) is

$$P_0 U - P (U - u)$$

This must equal the change in kinetic energy plus the change in internal energy of the gas. The former is

$$\frac{1}{2} m [(U - u)^2 - U^2]$$

and the latter is

$$m(E - E_0)$$

where  $E$  and  $E_0$  are the internal energies of the gas on the left and right sides of the shock front respectively and  $m$  is the mass of air crossing the unit cross section of the front per second and is given by

$$m = \rho_0 U = \rho (U - u)$$

Hence we have

$$P_0 U - P(U - u) = \frac{1}{2} m [(U - u)^2 - U^2] + m (E - E_0)$$

dividing by  $m$  we have

$$\frac{P_0}{\rho_0} - \frac{P}{\rho} = \frac{1}{2} [(U - u)^2 - U^2] + E - E_0$$

or

$$\frac{P_0}{\rho_0} + \frac{1}{2} U^2 + E_0 = \frac{P}{\rho} + \frac{1}{2} (U - u)^2 + E \quad \text{conservation of energy (III:3)}$$

For an ideal gas the internal energy may be calculated as follows:

$$P = k \rho^\gamma \quad \text{where } \rho = \frac{1}{v}$$

$$\begin{aligned} \text{Hence } E &= \int_{\frac{1}{\rho}}^{\infty} P d\left(\frac{1}{\rho}\right) = -k \int_{\frac{1}{\rho}}^{\infty} \rho^\gamma \frac{d\rho}{\rho^2} = -k \int_{\frac{1}{\rho}}^{\infty} \rho^{\gamma-2} d\rho \\ &= \frac{k \rho^{\gamma-1}}{\gamma-1} \quad k = \frac{P}{\rho^\gamma} \end{aligned}$$

therefore

$$E = \frac{1}{\gamma-1} \frac{P}{\rho}$$

and

$$E_0 = \frac{1}{\gamma-1} \frac{P_0}{\rho_0}$$

Substituting these values for  $E$  and  $E_0$  into equation (III.3) we obtain

$$\frac{P_0}{\rho_0} + \frac{1}{\gamma-1} \frac{P_0}{\rho_0} + \frac{1}{2} U^2 = \frac{P}{\rho} + \frac{1}{\gamma-1} \frac{P}{\rho} + \frac{1}{2} [U - u]^2$$

or

$$\frac{\gamma}{\gamma-1} \frac{P_0}{\rho_0} + \frac{1}{2} U^2 = \frac{\gamma}{\gamma-1} \frac{P}{\rho} + \frac{1}{2} [U - u]^2 \quad (\text{III.3a})$$

Now we may use these fundamental relations to derive the relation between the velocity of the shock wave and its pressure together with certain valuable information concerning the gases behind the shock. From equations (III.1) and (III.2) it follows that

$$P - P_0 = m [U - (U - u)]$$

hence

$$-\frac{1}{2} (P - P_0) [U + (U - u)] = \frac{m}{2} [(U - u)^2 - U^2]$$

and from the third equation (III.3) we have

$$P_0 U - P(U - u) - \frac{m}{2} [(U - u)^2 - U^2] = m (E - E_0)$$

substituting from above we have

$$P_0 U - P(U - u) + \frac{1}{2} (P - P_0) [U + (U - u)] = m (E - E_0)$$

which when cleared becomes

$$\frac{1}{2} (P + P_0) \frac{U - (U - u)}{m} = E - E_0 \quad \text{but } m = \rho_0 U = \rho (U - u)$$

which then is

$$\frac{1}{2} (P + P_0) \left[ \frac{1}{\rho_0} - \frac{1}{\rho} \right] = E - E_0 = \frac{1}{\gamma-1} \left( \frac{P}{\rho} - \frac{P_0}{\rho_0} \right) \quad (\text{III.4})$$

This equation may be interpreted as saying that the increase of internal energy across the shock front is due to the work done by the mean pressure in performing the compression.

From equation (III.4) we may solve for  $\frac{\rho}{\rho_0}$  which we do by multiplying the expression by  $\rho$  and dividing by  $P_0$  then

$$\frac{1}{2}(1 + \frac{P}{P_0})(\frac{\rho}{\rho_0} - 1) = \frac{1}{\gamma-1} (\frac{P}{P_0} - \frac{\rho}{\rho_0})$$

Let  $\frac{P}{P_0} = y$

and  $\frac{\rho}{\rho_0} = x$

Then it may be written

$$\frac{1}{2}(y + 1)(x - 1) = \frac{1}{\gamma-1} (y - x)$$

the solution of which is

$$x = \frac{\gamma-1 + (\gamma+1)y}{\gamma+1 + (\gamma-1)y} = \frac{\rho}{\rho_0} \quad \text{ratio of densities (III.5)}$$

From equation (III.1) we have

$$U - u = \frac{\rho_0}{\rho} U$$

Hence equation (III.2) may be written as

$$P + \frac{\rho_0^2}{\rho} U^2 = P_0 + \rho_0 U^2 \quad \text{or} \quad \rho_0 U^2 (1 - \frac{\rho_0}{\rho}) = P - P_0$$

or

$$U^2 = \frac{P - P_0}{\rho_0 (1 - \frac{\rho_0}{\rho})} = \frac{P_0}{\rho_0} \frac{(y - 1)}{1 - \frac{1}{x}}$$

Let  $\frac{\gamma P_0}{\rho_0} = a_0^2$  where  $a_0$  is the velocity of sound in the undisturbed medium.

$$\therefore \frac{U^2}{a_0^2} = \frac{1}{\gamma} \frac{(y - 1)}{1 - \frac{1}{x}} \quad \text{and substituting from above we have}$$

$$\frac{U^2}{a_0^2} = \frac{1}{\gamma} \frac{y - 1}{1 - \frac{\gamma+1 + (\gamma-1)y}{\gamma-1 + (\gamma+1)y}} = \frac{1}{2\gamma} [\gamma-1 + (\gamma+1)y]$$

Therefore the first of the important derived relations is the velocity pressure relationship for the shock wave

$$\frac{U^2}{a_0^2} = \frac{1}{2\gamma} [\gamma-1 + (\gamma+1)y] \quad \text{velocity of shock wave (III.6)}$$

From equation (III.1) we have

$$u = (1 - \frac{\rho_0}{\rho}) U$$

hence  $\frac{u}{U} = 1 - \frac{\rho_0}{\rho} = 1 - \frac{\gamma+1 + (\gamma-1)y}{\gamma-1 + (\gamma+1)y}$

or  $\frac{u}{U} = \frac{2(\gamma-1)}{\gamma-1 + (\gamma+1)y}$  ratio of particle velocity to shock velocity. (III.7)

If  $c = \sqrt{\frac{\gamma P}{\rho}}$  where  $c$  is the velocity of sound behind the shock front

$$\frac{c^2}{a_c^2} = \gamma \frac{P}{\rho} \cdot \frac{\rho_0}{\gamma P_0} = \frac{P}{P_0} \cdot \frac{\rho_0}{\rho}$$

$$\frac{c^2}{a_c^2} = \gamma \left[ \frac{\gamma+1 + (\gamma-1)y}{\gamma-1 + (\gamma+1)y} \right] \text{ ratio of velocities of sound behind and in front of shock wave. (III.8)}$$

From equation (III.6) and III.7) we can derive the ratio of the particle velocity behind the shock front to the speed of sound in front of shock

$$\frac{u^2}{a_c^2} = \frac{2(\gamma-1)^2}{\gamma[\gamma-1 + (\gamma+1)y]} \quad (III.9)$$

These equations although derived for a shock wave moving into still air,  $u_0 = 0$ , will hold for a shock wave moving into air traveling with a uniform velocity  $u_0$ , if we understand  $U$  to mean the velocity of the front relative to the moving air.

If the medium into which the shock waves travel is air which has a  $\gamma$  equal to 1.4 these equations may be simplified and rewritten using

$$z = \frac{P}{P_0} = \gamma - 1$$

$$\frac{\rho}{\rho_0} = \frac{1 + 6y}{6 + y} = \frac{7 + 6z}{7 + z} \quad (III.5a)$$

$$\frac{U^2}{a_0^2} = \frac{1 + 6y}{7} = 1 + \frac{6}{7} z \quad (III.6a)$$

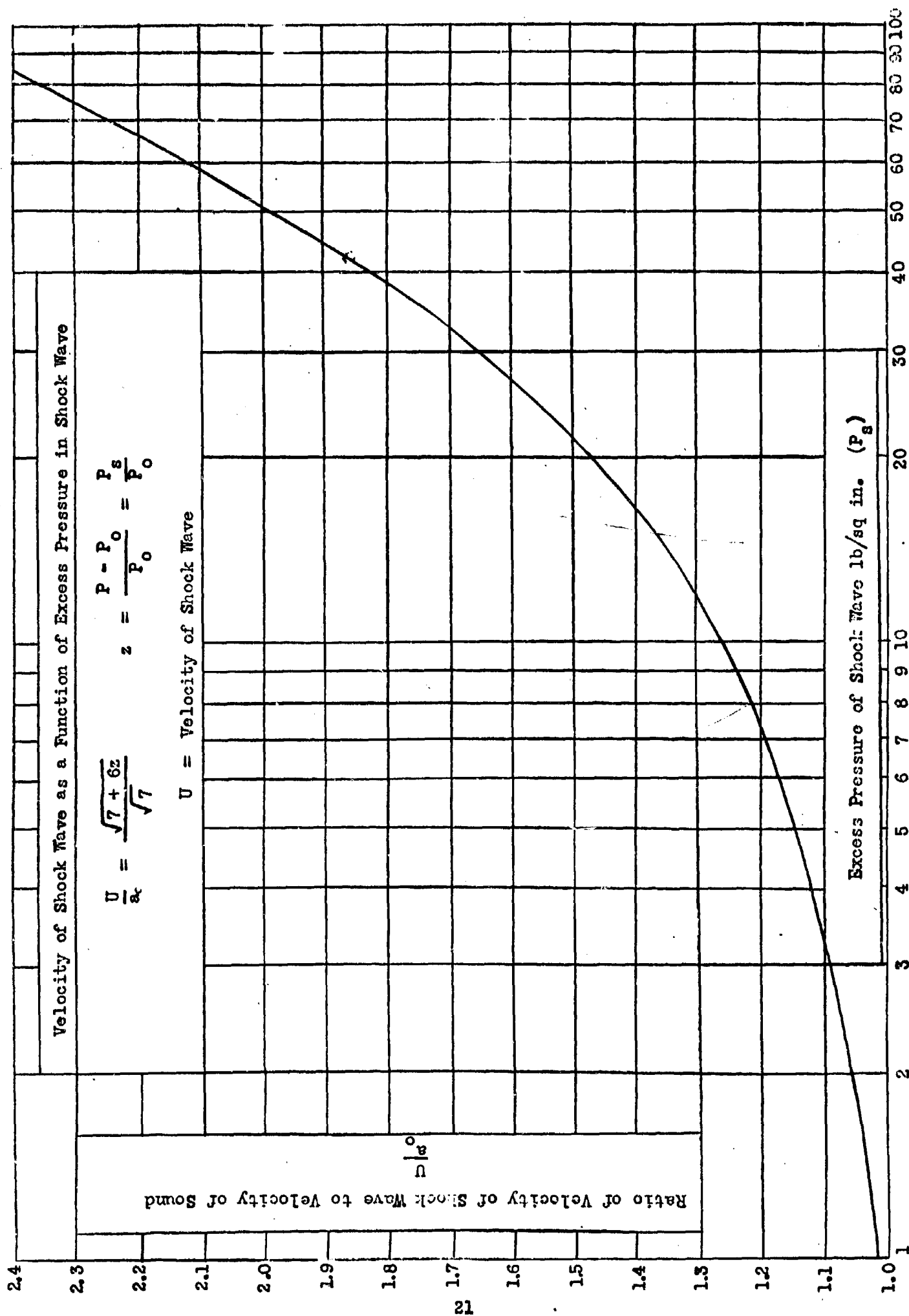
$$\frac{u}{U} = \frac{5(\gamma-1)}{1 + 6y} = \frac{5z}{7 + 6z} \quad (III.7a)$$

$$\frac{c^2}{a_0^2} = \gamma \left( \frac{6 + y}{1 + 6y} \right) = (z + 1) \left( \frac{7 + z}{7 + 6z} \right) \quad (III.8a)$$

$$\frac{u^2}{a_0^2} = \frac{25(y-1)^2}{7(1+6y)} = \frac{25x^2}{7(7+6x)} \quad (\text{III.9a})$$

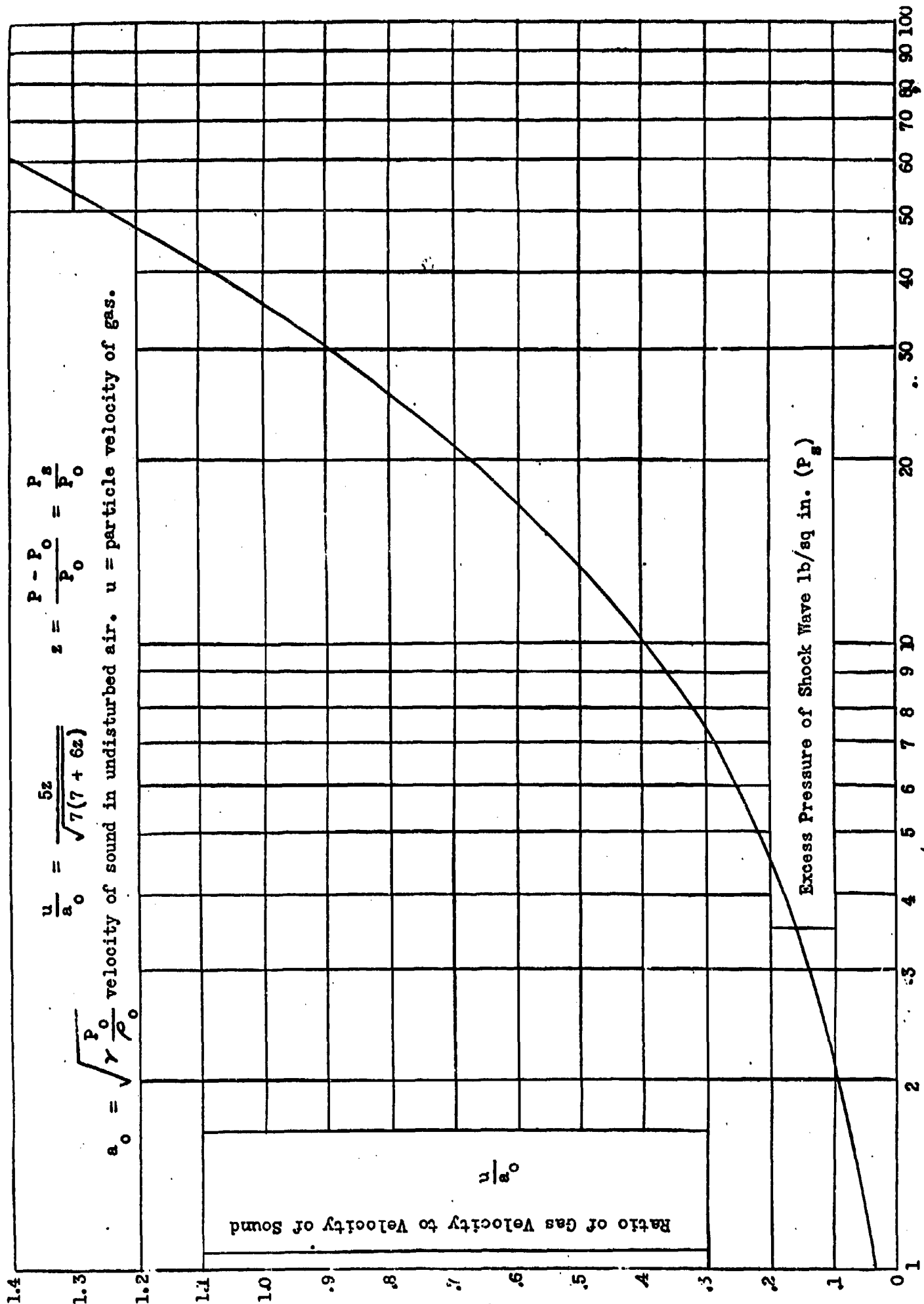
Three of these quantities are plotted in the following pages (Graphs 1, 2, and 3) as functions of the excess pressure behind the shock in pounds per square inch. The excess pressure is  $(P - P_0)$  where  $P_0$  is assumed to be 14.7 pounds for computational purposes.

Graph 1

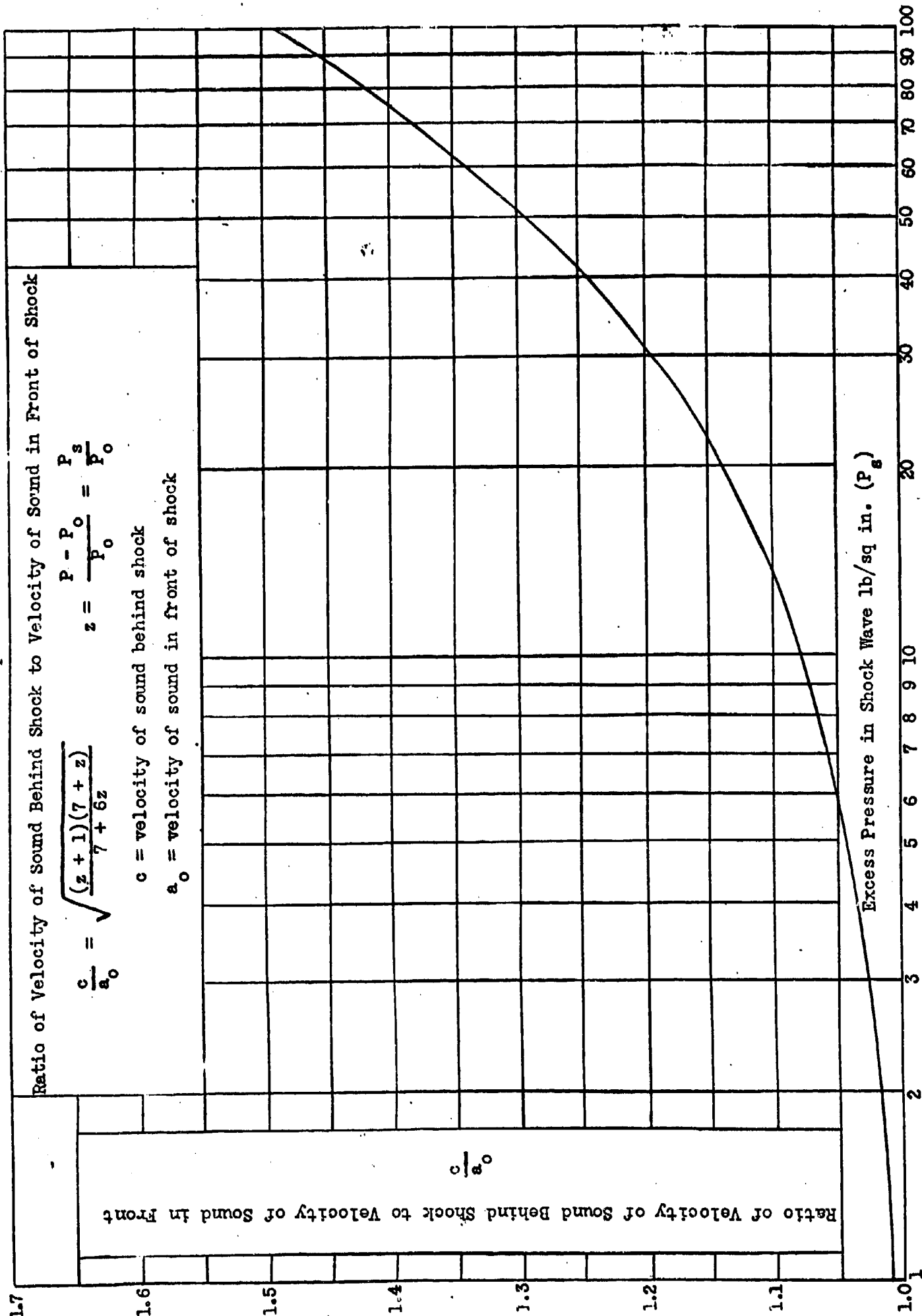


Graph 2

Graph 2

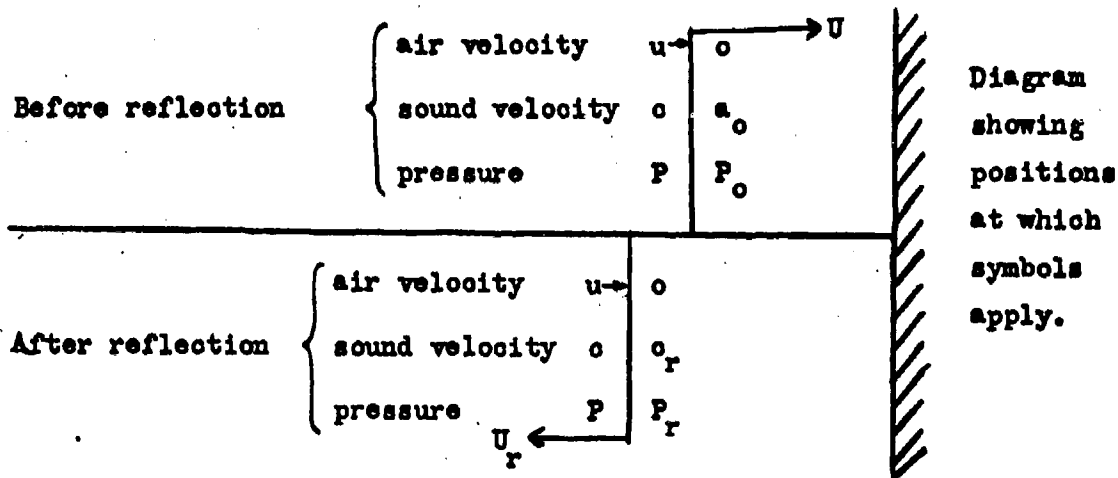


Graph 3



#### IV. REFLECTION OF SHOCK WAVES

When a shock wave strikes perpendicularly on a flat surface, a shock wave is reflected. The pressure at the surface jumps instantaneously from the atmospheric pressure  $P_0$  to the pressure  $P_r$  behind the reflected shock wave. At a point close in front of the reflecting plane the pressure first changes to  $P$  the pressure behind the incident shock wave and then to  $P_r$  as the reflected wave reaches it. The relationship between  $y_r = P_r/P_0$  and  $y = P/P_0$  can be derived from the shock wave equations.



If  $u$  is the velocity of the air behind the incident shock wave,  $U$  the velocity of propagation in still air,  $a_0$  the velocity of sound in the undisturbed air,  $U_r$  the velocity of propagation of the reflected wave and  $a_r$  the velocity of sound in air behind the incident wave, the shock wave equations for the incident wave are:

$$\frac{u}{U} = \frac{2(y-1)}{\gamma-1 + (\gamma+1)y} \quad (\text{IV.1})$$

$$\frac{U^2}{a_0^2} = \frac{1}{2\gamma} [\gamma-1 + (\gamma+1)y] \quad (\text{IV.2})$$

$$\frac{a_r^2}{a_0^2} = y \left[ \frac{\gamma+1 + (\gamma-1)y}{\gamma-1 + (\gamma+1)y} \right] \quad (\text{IV.3})$$

$$\frac{u^2}{a_0^2} = \frac{2(y-1)^2}{\gamma[\gamma-1 + (\gamma+1)y]} \quad (\text{IV.4})$$

The reflected shock wave advances with a velocity  $u + U_r$  relative to air in which the velocity of sound is  $a_r$ . The velocity of the air behind the reflected wave relative to that in front is  $u$  as in the

incident wave; thus the equations for the reflected wave are:

$$\frac{u}{u + u_r} = \frac{2(y_r - 1)}{\gamma - 1 + (\gamma + 1)y_r} \quad (\text{IV.5})$$

then

$$\frac{u}{u_r} = \frac{2(y_r - 1)}{\gamma + 1 + (\gamma - 1)y_r} \quad (\text{IV.5a})$$

$$\frac{c_r^2}{c^2} = y_r \left[ \frac{\gamma + 1 + (\gamma - 1)y_r}{\gamma - 1 + (\gamma + 1)y_r} \right] \quad (\text{IV.6})$$

and in reflected wave

$$\frac{u^2}{c_r^2} = \frac{2}{\gamma} \frac{(y_r - 1)^2}{(\gamma - 1) + (\gamma + 1)y_r} \quad (\text{IV.7})$$

Here  $c_r$  is the velocity of sound in the air behind the reflected wave. The relationship between  $y$  and  $y_r$  is found by eliminating  $u$ ,  $a$ , and  $c$  between equations (IV.3), (IV.4), and (IV.7). From (IV.3), and (IV.4) we have

$$\frac{u^2}{c^2} = \frac{2}{\gamma} \frac{(y - 1)^2}{y[\gamma + 1 + (\gamma - 1)y]} \quad (\text{IV.7a})$$

Equating this to (IV.7) we have

$$\frac{(y - 1)^2}{y[\gamma + 1 + (\gamma - 1)y]} = \frac{(y_r - 1)^2}{\gamma - 1 + (\gamma + 1)y_r} \quad (\text{IV.8})$$

If one assumes  $\gamma$  for air equal to 1.4 then equation (IV.8) becomes

$$\frac{(y - 1)^2}{y(6 + y)} = \frac{(y_r - 1)^2}{1 + 6 y_r} \quad (\text{IV.8a})$$

This can be expanded into

$$y^2 y_r^2 - 8 y_r y^2 + 6 y y_r^2 + 8 y = 6 y_r + 1$$

and factored

$$8 y(1 - y y_r) - 6 y_r(1 - y y_r) = 1 - y^2 y_r^2$$

which equals

$$8 y - 6 y_r = 1 + y y_r \quad (\text{IV.8b})$$

Let  $P = P_s + P_o$  then  $y = \frac{P_s + P_o}{P_o}$

$P_r = P_f + P_o$   $y_r = \frac{P_f + P_o}{P_s + P_o}$

After substituting into (IV.8b) and manipulating one arrives at the relationship

$$\frac{P_f}{P_s} = 2 \left( \frac{7P_o + 4P_s}{7P_o + P_s} \right) \quad (\text{IV.9})$$

This is the relationship between the reflected excess pressure over atmospheric and the incident excess pressure over atmospheric. This is the same as the relationship between gauge pressures measured face on and side on to the blast wave.

It is apparent that for weak shocks where  $P_s \rightarrow 0$

$$\frac{P_f}{P_s} \rightarrow 2$$

and for very strong shocks where  $P_s \rightarrow \infty$

$$\frac{P_f}{P_s} \rightarrow 8$$

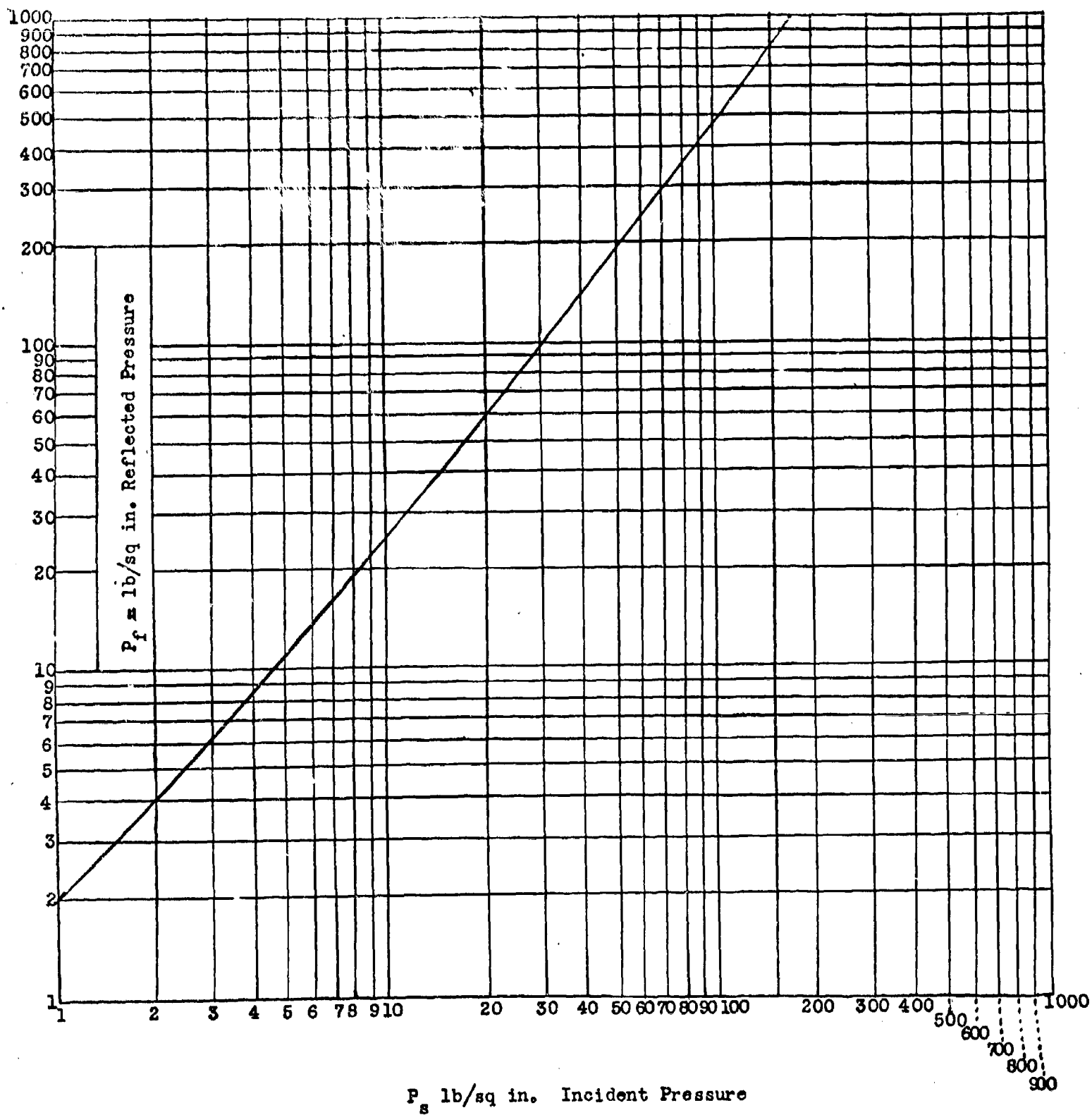
Pressures for intermediate strength shocks are plotted on the next page. (Graph 4)

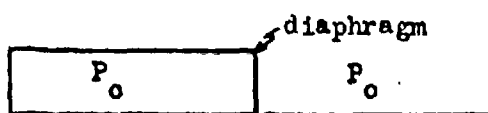
## V. THEORY OF SHOCK WAVE FORMATION IN A TUBE FROM A BURSTING DIAPHRAGM

Consider a tube of constant cross section closed at one end, with a gas-tight diaphragm fixed at some point in the tube in such a way that a section of the tube bounded by the closed end and the diaphragm may be pumped up to a pressure  $P_c$ , while the remainder of the tube remains at a pressure  $P_o$ . Then if the diaphragm is suddenly broken by air pressure or other means a shock wave will be formed in the low pressure section of the tube advancing along the tube away from the diaphragm. At the same time a rarefaction wave will be formed in the high pressure section of the tube which will progress back into the high pressure gas until it is reflected at the closed end of the compression chamber. Meanwhile the shock wave will progress down the tube until it is reflected with either positive or negative phase at the other end of the tube. The sequence of events may be shown in the following way:

Graph 4

Pressures in Incident and Reflected Shock Waves

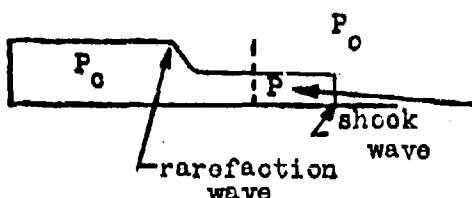




before breaking



at breaking



after breaking  
region in which pressure is  $P$ , velocity of gas is  $u$ , velocity of sound is  $a$  and velocity of shock wave is  $U$ .

From the Rankine-Hugoniot equations we have the relationship between the particle velocity behind the shock front and the pressure ratio  $P/P_0$  at the shock front. This is:

$$\frac{u^2}{a_0^2} = \frac{2}{\gamma} \frac{(\gamma - 1)^2}{\gamma - 1 + (\gamma + 1)y} \quad (V.1)$$

where  $y = \frac{P}{P_0}$ ,  $a_0 = \sqrt{\gamma \frac{P_0}{\rho_0}}$ , and  $u$  = particle velocity behind shock front.

The region behind the diaphragm is propagating a rarefaction wave because of the relief of pressure by the bursting of the diaphragm. The velocity of the particles in the rarefaction wave where the pressure is  $P$  is given by

$$u = \pm \int_{P_0}^P \frac{a}{\rho} \frac{d\rho}{dP} \quad \text{which has been evaluated in equation (I.9), and found to be}$$

$$u = \pm \frac{2}{\gamma - 1} a_1 \left[ 1 - \left( \frac{P}{P_0} \right)^{\frac{\gamma - 1}{2\gamma}} \right] \quad (V.2)$$

where  $P_0$  is the pressure of the region into which the wave advances and  $a_1$  is the velocity of sound in this region.

Now  $P = P_0 \frac{P}{P_0} = P_0 y$

so equation (V.2) becomes

$$u = \pm \frac{2}{\gamma - 1} a_1 \left[ 1 - \left( y \frac{P_0}{P_0} \right)^{\frac{\gamma - 1}{2\gamma}} \right] \quad (V.2a)$$

These two particle velocities must be equal (at the diaphragm after breaking); otherwise a local region of vacuum or high pressure will develop in time. So, equating the two particle velocities we have

$$\frac{2a_0^2 (\gamma - 1)^2}{\gamma [\gamma - 1 + (\gamma + 1)y]} = \frac{4a_1^2}{(\gamma - 1)^2} \left[ 1 - \left( y \frac{P_0}{P_0} \right)^{\frac{\gamma - 1}{2\gamma}} \right]^2 \quad (V.3)$$

Assume  $a_0 = a_1$  which means that sufficient time must elapse after pumping up the compression chamber for the gas to be reduced to ambient temperature. Extracting the square root on both sides we have then

$$\frac{\sqrt{2(y-1)}}{\sqrt{\gamma[\gamma-1+(\gamma+1)y]}} = \frac{2}{\gamma-1} \left[ 1 - \left(y \frac{P_0}{P_c}\right)^{\frac{\gamma-1}{2\gamma}} \right]$$

Assume  $\gamma = 1.4$ , then

$$\frac{y-1}{\sqrt{7(1+6y)}} = 1 - \left(y \frac{P_0}{P_c}\right)^{1/7}$$

$$\text{of } \frac{P_0}{P_c} = \left[ \frac{y}{1 - \frac{y-1}{\sqrt{7(1+6y)}}} \right]^{1/7} \quad \text{Implicit solution for } y \quad (V.4)$$

This equation then gives the pressure in the compression chamber before breaking the diaphragm necessary to establish a shock wave of pressure  $P$  travelling down the tube. This relationship is plotted on the following page. (Graph 5). On the second page following is plotted also the values of excess chamber pressure over atmospheric and the excess shock wave pressure over atmospheric in pounds per square inch. (Graph 6).

The compression chamber pressure may also be evaluated in terms of a given Mach number  $M$  behind the shock front.

$$\text{Since } \frac{u}{c} = M = \frac{5z}{\sqrt{7(7+6z)}} = \frac{5(y-1)}{\sqrt{7(1+6y)}} \quad \text{from equation (III.9a)} \quad (V.5)$$

Equation (V.4) may be rewritten as

$$\frac{P_c}{P_0} = \frac{y}{\left(1 - \frac{M}{5}\right)^7} = \frac{z+1}{\left(1 - \frac{M}{5}\right)^7} \quad (V.6)$$

Solving (V.5) for  $z$  in terms of  $M$ ,

$$z = 0.84 M^2 + 1.4 M \sqrt{1 + 0.36 M^2}$$

and

$$\frac{P_c}{P_0} = \frac{1 + 0.84 M^2 + 1.4 M \sqrt{1 + 0.36 M^2}}{\left(1 - \frac{M}{5}\right)^7} \quad (V.7)$$

$M$	$P_c/P_0$	$P_s/P_0$
1.4	52	4.25
1.2	29.3	3.3
1	16.5	2.5

Graph 5  
 Compression Chamber Pressure in Atmospheres  
 as a Function of  
 Shock Wave Pressure in Atmospheres

50  
 40  
 30  
 20  
 10  
 9  
 8  
 7  
 6  
 5  
 4  
 3  
 2  
 1

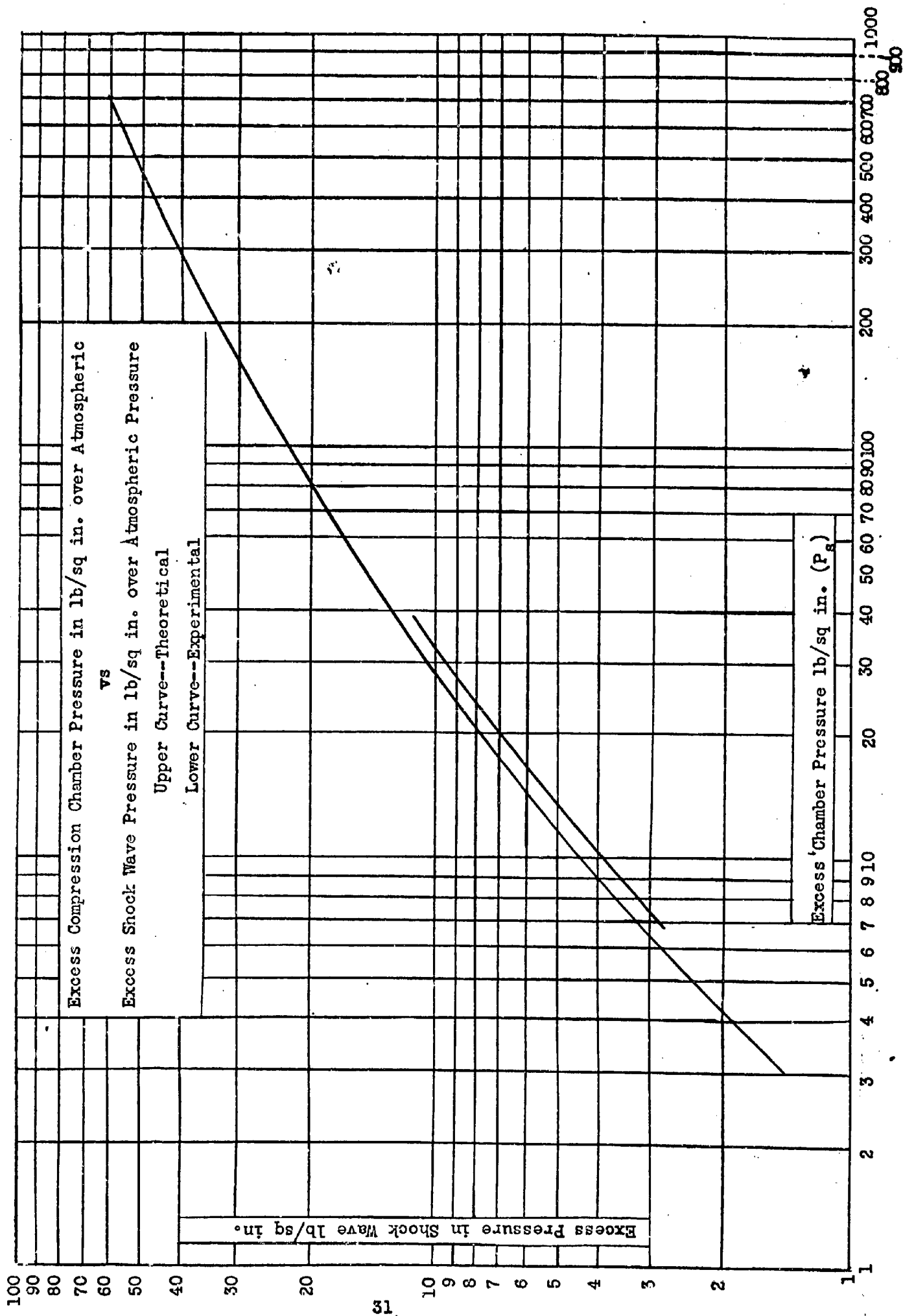
Chamber Pressure in Atmospheres =  $P_c$

$$\frac{P_c}{P_o}$$

Shock Wave Pressure in Atmospheres

$$y = \frac{P}{P_o}$$

Graph 6



In case gases of different density and  $\gamma$  are used in compression chamber and expansion chamber the relationship between compression chamber pressure and shock strength will be different. If the gas in the compression chamber has a gamma  $\gamma_1$  and sound velocity  $a_1$  and the gas in the expansion chamber has a gamma  $\gamma_2$  and sound velocity  $a_2$  then it can be shown that

$$\frac{P_o}{P_o} = \frac{y}{\left[ 1 - \frac{a_2}{a_1} \frac{(y-1)(\gamma_1-1)}{\sqrt{2\gamma_2(\gamma_2-1 + (\gamma_2+1)y)}} \right]^{\frac{2\gamma_1}{\gamma_1-1}}} \quad (V.8)$$

For example, if helium is used in the compression chamber and air in the expansion chamber,

$$\gamma_1 = 1.66$$

$$\gamma_2 = 1.40$$

$$\frac{a_1}{a_2} = 1.93$$

At  $y = 3$ ,

$$\frac{P_o}{P_o} = \frac{3}{\left[ 1 - \frac{.518 \times 2 \times 0.66}{\sqrt{2.80 (0.40 + 2.40 \times 3)}} \right]^{5.04}} = 6.7$$

For comparison, air in both chambers give  $\frac{P_o}{P_o} = 11.7$  for  $y = 3$ .

## VI. ADIABATIC THEORY OF THE SHOCK TUBE

An explicit expression for  $P$  can be obtained from an adiabatic solution of the bursting diaphragm problem which is approximately correct and is convenient for calculational purposes. The errors in the approximation are indicated in tabular form at the end of this section.

The line of reasoning is the same in this case as in the previous more exact solution but the expression for the particle velocity behind an adiabatic compression wave is substituted for that behind a shock wave.

We have from equation (I.8b) that the particle velocity behind a compressional wave of pressure  $P$  is:

$$u = \frac{2a_0}{\gamma-1} \left[ \left( \frac{P}{P_0} \right)^{\frac{\gamma-1}{2\gamma}} - 1 \right] \quad \text{particle velocity be- (VI.1)} \\ \text{hind compression wave}$$

and from equation (I.9) the particle velocity behind a rarefaction wave advancing into a region of pressure  $P_0$  and velocity of sound  $a_1$  is:

$$u = \frac{2a_1}{\gamma-1} \left[ 1 - \left( \frac{P}{P_0} \right)^{\frac{\gamma-1}{2\gamma}} \right] \quad \text{(VI.2)}$$

Assume  $a_1 = a_0$  as before and equate the particle velocities giving

$$1 - \left( \frac{P}{P_0} \right)^{\frac{\gamma-1}{2\gamma}} = \left( \frac{P}{P_0} \right)^{\frac{\gamma-1}{2\gamma}} - 1 \quad \text{(VI.3)}$$

but 
$$\frac{P}{P_0} = \frac{P}{P_0} \cdot \frac{P_0}{P_c} = y \frac{P_0}{P_c}$$

so 
$$1 - \left( y \frac{P_0}{P_c} \right)^{\frac{\gamma-1}{2\gamma}} = y \frac{\gamma-1}{2\gamma} - 1 \quad \text{(VI.3a)}$$

or 
$$\frac{P_c}{P_0} = \frac{y}{\left( 2 - y \frac{\gamma-1}{2\gamma} \right)^{\frac{2\gamma}{\gamma-1}}} \quad \text{(VI.4)}$$

and likewise the explicit solution for  $y$  is

$$y = \frac{(2)^{\frac{2\gamma}{\gamma-1}} P_c / P_0}{\left[ \left( \frac{P_c}{P_0} \right)^{\frac{\gamma-1}{2\gamma}} + 1 \right]^{\frac{2\gamma}{\gamma-1}}} \quad \text{(VI.5)}$$

If  $\gamma = 1.4$  (air) and we let  $P_o/P_o = \omega$ , then

$$y = \frac{2^7 \omega}{(\omega^{1/7} + 1)^7} \quad \text{an explicit expression for } y \text{ (VI.5a) in terms of } \omega.$$

$$y = \frac{128 \omega}{(\omega^{1/7} + 1)^7} \quad \text{(VI.5b)}$$

also in terms of the same variable

$$\omega = \frac{y}{(2 - y^{1/7})^7} \quad \text{(VI.4a)}$$

The results of equation (V.4) give

$$\omega = \frac{y}{\left[1 - \frac{(y-1)}{\sqrt{7(1+6y)}}\right]^7} \quad \text{and} \quad \omega^{1/7} = \frac{y^{1/7}}{1 - \frac{(y-1)}{\sqrt{7(1+6y)}}}$$

Substituting this value of  $\omega$  into equation (VI.5a) and extracting the root, we have

$$y_a^{1/7} = \frac{2 y^{1/7}}{1 + y^{1/7} - \frac{(y-1)}{\sqrt{7(1+6y)}}}$$

so that

$$\frac{y_a}{y} = \frac{128}{\left[1 + y^{1/7} - \frac{(y-1)}{\sqrt{7(1+6y)}}\right]^7} \quad \text{Ratio of amplitudes calculated for the compressional wave under conditions of adiabatic and shock wave formation. (VI.6)}$$

These are tabulated as functions of  $y$

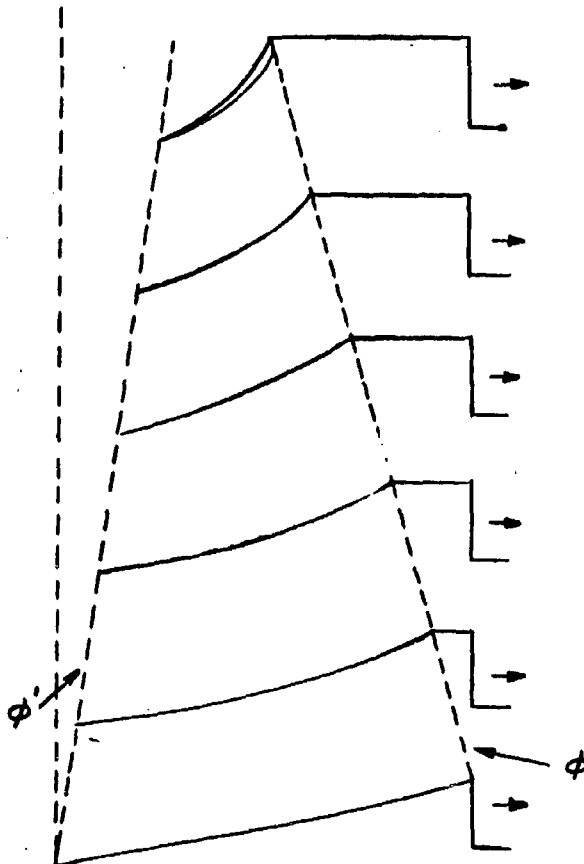
$y$	$\frac{y_a}{y}$	$\frac{(y_a - y)}{y} \times 100$
1	1.00	0
2	1.01	1
3	1.02	2
4	1.033	3.3
5	1.05	5.0
6	1.07	7.0
7	1.09	9.0
8	1.115	11.5
9	1.14	14.0
10	1.17	17.0

This table gives an idea of the range of shock pressures over which the adiabatic expression for  $y$  may be used without too great an error in results.

## VII. THE PRODUCTION OF ARTIFICIAL BLAST WAVES

(The calculation of the relative dimensions of the tube to produce shock waves with a peaked top.)

The shape of the shock waves produced by the bursting of a diaphragm in a tube will change as the wave progresses down the tube as shown in the following series of sketches.



$$\tan \phi = \frac{5(y-1) + \sqrt{7y(6+y)}}{1 + 6y} - 1$$

$$\tan \phi' = \frac{-(6+y) - (6/y^{1/7} - 1)\sqrt{7y(6+y)}}{1 + 6y}$$

Examples:  $y = 2$

$$\phi = 11.3^\circ$$

$$\phi' = -14.3^\circ$$

angle of  $P = P_0$   
in the trough.

It is obvious that at some point down the tube the rarefaction tip will catch up with the shock front and a peaked wave will be produced. It is the position of this point which will be sought in the analysis. The sequence of events in the tube after the diaphragm has broken consists of the production of a shock wave traveling down the expansion tube with a velocity  $U$ , followed by a temperature discontinuity traveling at a velocity  $u$ , and a rarefaction wave traveling back into the compression chamber. The initial part or tip of the rarefaction wave travels back with the velocity of sound  $b_0$  in the compression chamber which is assumed to be equal to  $a_0$ . This tip is then reflected from the closed end of the compression chamber and travels for a short distance through a variable density region until it reaches

the gas of constant pressure P. It then travels down the compression chamber and expansion tube with a constant velocity  $b_1$  until it reaches the boundary between the cool gas that was originally in the compression chamber and the gas passed over by the shock wave which has been compressed and heated. This boundary travels with the particle velocity  $u$  behind the shock front. When the rarefaction tip reaches this boundary it proceeds with a slightly greater velocity  $b_2$  until it eventually catches up with the shock front.

The various velocities with which the different sections of the shock wave travel in the tube are derived below:

$$b_0 = a_0 \quad (\text{VII.1})$$

where  $b_0$  is the velocity of the tip into the compression chamber. After reflection from the closed end of the compression chamber and after reaching the constant pressure region P the velocity of the rarefaction tip is

$$b_1 = u + c_1 \quad (\text{VII.2})$$

where  $u = \frac{5a_0 (y - 1)}{\sqrt{7(1 + 6y)}}$  Equation (III.9a)

and  $c_1 = a_0 \left(\frac{P}{P_0}\right)^{1/7} = a_0 \left(1 - \frac{(y - 1)}{\sqrt{7(1 + 6y)}}\right)$  Equation (V.4)

so that

$$b_1 = a_0 \left(1 + \frac{4(y - 1)}{\sqrt{7(1 + 6y)}}\right) \text{ velocity of tip in cool gas.} \quad (\text{VII.2a})$$

The velocity of the tip after passing the gas boundary is

$$b_2 = u + c \quad (\text{VII.3})$$

where  $u$  is the same as above and

$$c = a_0 \sqrt{\frac{y(6 + y)}{(1 + 6y)}} \quad \text{Equation (III.8a)}$$

so that

$$b_2 = a_0 \left[ \frac{5(y - 1) + \sqrt{7y(6 + y)}}{\sqrt{7(1 + 6y)}} \right] \text{ velocity of tip in hot gas.} \quad (\text{VII.3a})$$

After breaking the diaphragm the velocity of the trough of the rarefaction wave back into the compression chamber will be  $v_0$  which is the local velocity of sound minus the particle velocity. So

$$v_0 = c_1 - u \quad (\text{VII.4})$$

so that

$$v_0 = a_0 \left[ 1 - \frac{6(y-1)}{\sqrt{7(1+6y)}} \right] \text{ velocity of trough toward the closed end of the compression chamber. (VII.4a)}$$

It may be noted that  $v_0 = 0$  when  $y = 2.88$  which means that the trough is stationary at this shock pressure. It moves toward the closed end if the pressure is less than  $2.88 P_0$  and moves toward the shock front if the shock pressure exceeds this value.

After reflection and after the tip of the rarefaction wave has moved out of the variable density region, the trough of the rarefaction wave has a velocity  $v_1$  which is the local velocity of sound in a gas cooled by adiabatic expansion from a pressure  $P_0$  to a pressure  $P_r$ . So

$$v_1 = c_2 \quad (\text{VII.5})$$

where

$$c_2 = a_0 \left( \frac{P_r}{P_0} \right)^{1/7}$$

$$\text{but } \left( \frac{P_r}{P_0} \right)^{1/7} = 2 \left( \frac{P}{P_0} \right)^{1/7} - 1 = 2 \left( \frac{P}{P_0} \right)^{1/7} \left( \frac{P_0}{P} \right)^{1/7} - 1 \text{ from equation (II.16)}$$

$$\text{and } \frac{P_0}{P} = \omega \quad \text{and } \frac{P}{P_0} = y$$

$$\text{so } c_2 = a_0 \left( \frac{2y^{1/7}}{\omega^{1/7}} - 1 \right)$$

$$\text{but } \omega^{1/7} = \frac{y^{1/7}}{1 - \frac{(y-1)}{\sqrt{7(1+6y)}}} \text{ from equation (V.4)}$$

so that

$$v_1 = a_0 \left[ 1 - \frac{2(y-1)}{\sqrt{7(1+6y)}} \right] \text{ velocity of the trough in cool gas. (VII.5a)}$$

After crossing the gas boundary the velocity of the trough is equal to the velocity of sound in a gas cooled from a pressure  $P$  and sound velocity  $c$  to a pressure  $P_{r2}$  and a sound velocity  $a_3$  plus the residual particle velocity which is no longer zero after crossing the gas boundary.

From section II we have the equations which give the value of  $P_{r2}$  after passing through the gas boundary as

$$\left( \frac{P_{r2}}{P} \right)^{1/7} = \frac{2a_1}{c + a_1} \left( \frac{P_r}{P} \right)^{1/7} + \frac{c - a_1}{c + a_1} \quad (\text{VII.6})$$

now  $a_1 = a_0 \left(\frac{P}{P_0}\right)^{1/7}$

and from adiabatic theory of the tube we have  $\left(\frac{P}{P_0}\right)^{1/7} = 2 - y^{1/7}$

so  $a_1 = a_0 (2 - y^{1/7})$

and  $c = a_0 \left[ \frac{y(6+y)}{1+6y} \right]^{1/2}$  from equation (III.8a)

From the theory of reflection of adiabatic waves (section II) we have

$$\left(\frac{P_{r1}}{P}\right)^{1/7} = 2 - \left(\frac{P_0}{P}\right)^{1/7} = \frac{3 - 2y^{1/7}}{2 - y^{1/7}} \quad \text{since } \left(\frac{P_0}{P}\right)^{1/7} = \frac{1}{2 - y^{1/7}}$$

Substituting these values we have then

$$\left(\frac{P_{r2}}{P}\right)^{1/7} = \frac{2a_0}{c + a_1} (3 - 2y^{1/7}) + \frac{c - a_1}{c + a_1} \quad (\text{VII.7})$$

So the velocity of the trough will then be

where  $u = \frac{5a_0(y-1)}{\sqrt{7(1+6y)}} = \frac{5c(y-1)}{\sqrt{7y(6+y)}}$  the particle velocity behind the shock wave

$u_1 = 5c \left[ 1 - \left(\frac{P_{r2}}{P}\right)^{1/7} \right]$  the particle velocity caused by the rarefaction wave

and  $a_3 = c \left(\frac{P_{r2}}{P}\right)^{1/7}$

Substituting values for these quantities we then have that the velocity  $v_2$  of the trough in the hot gas is

$$v_2 = 5c \left\{ \frac{y-1}{\sqrt{7y(6+y)}} - 1 + \frac{6}{5} \left( \frac{2a_0(3 - 2y^{1/7}) + c - a_1}{c + a_1} \right) \right\} \quad (\text{VII.8})$$

The position of the shock wave in the tube at any time may be graphically portrayed in a chart which shows the locus of the various points as a function of time and distance down the tube. If distance along the tube from the diaphragm divided by the length of the compression chamber is plotted as  $x$  along the abscissa and if the quantity  $a_0 t/L_0$  is plotted along the ordinate, then straight lines may be drawn (representing the locus of the points as a function of time)

with the position of the diaphragm as the origin and at angles  $\theta_1, \theta_2$ , etc. with the abscissa. The tangents of the angles  $\theta_1, \theta_2$ , etc., are the inverse functions of the velocities of the points. We then have defined for the various components representing shock velocity, trough velocity, rarefaction tip velocity, etc., the tangents of the angles of their loci as follows:

$$\tan \theta_1 = \frac{a_0}{U} \quad \text{shock front} \quad (\text{VII.9})$$

$$\tan \theta_2 = \frac{a_0}{v_0} \quad \text{gas boundary} \quad (\text{VII.10})$$

$$\tan \theta_3 = \frac{a_0}{b_0} \quad \text{rarefaction tip in compression chamber} \quad (\text{VII.11})$$

$$\tan \theta_4 = \frac{a_0}{v_1} \quad \text{rarefaction tip in cool gas} \quad (\text{VII.12})$$

$$\tan \theta_5 = \frac{a_0}{v_2} \quad \text{rarefaction tip in hot gas} \quad (\text{VII.13})$$

$$\tan \theta_6 = \frac{a_0}{v_0} \quad \text{trough into compression chamber} \quad (\text{VII.14})$$

$$\tan \theta_7 = \frac{a_0}{v_1} \quad \text{trough in cool gas} \quad (\text{VII.15})$$

$$\tan \theta_8 = \frac{a_0}{v_2} \quad \text{trough in hot gas} \quad (\text{VII.16})$$

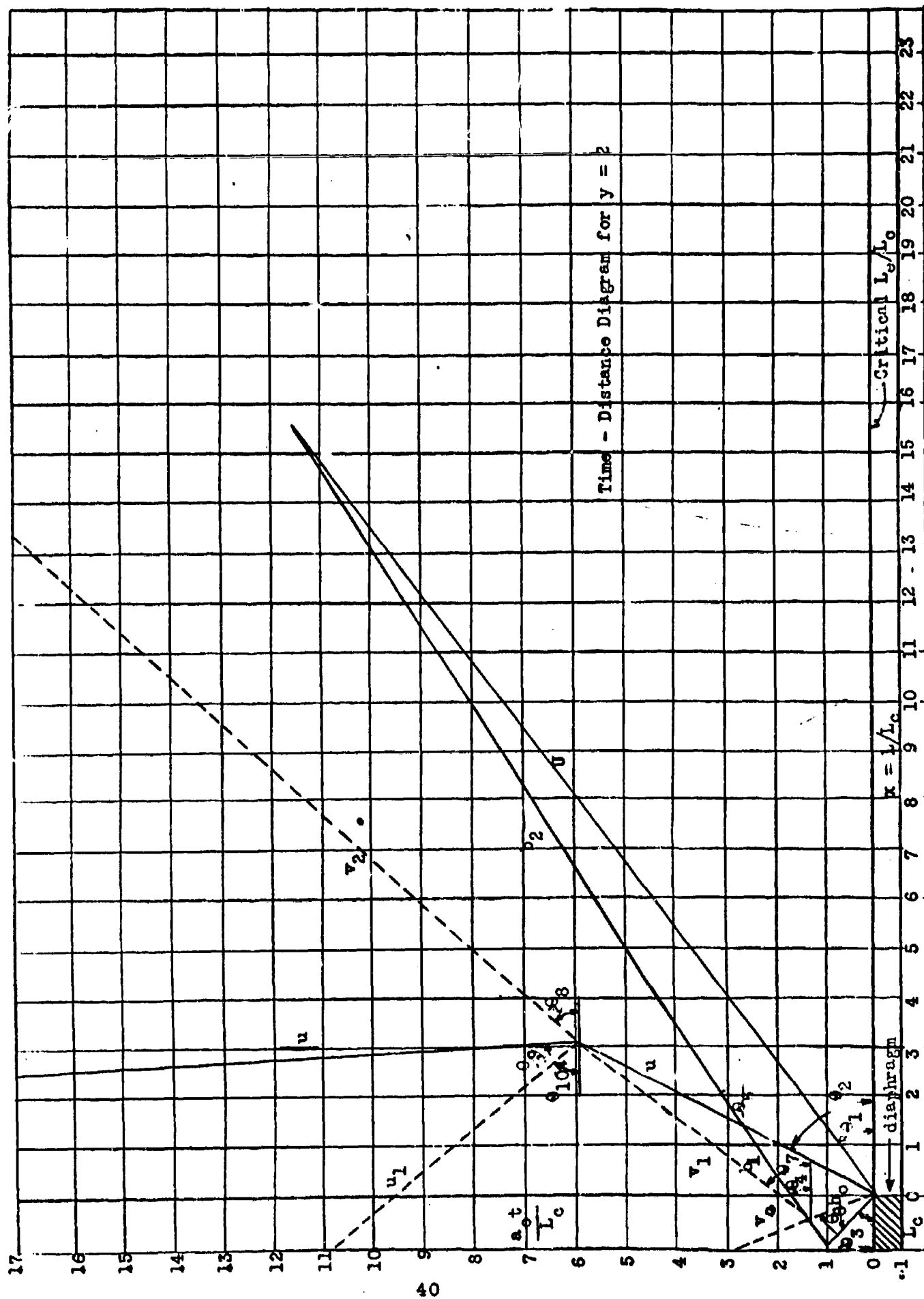
A time distance chart for a shock wave having a pressure ratio  $y = 2$  is shown on a following page. (Graph 7.) From it a great deal of information may be gleaned. For example, the length of the expansion tube necessary to insure that the shock wave have a pointed top like a blast wave can be found at the intersection of the locus of the shock front (line U) and the locus of the rarefaction tip (line  $b_2$ ). In this case it appears that the length of the expansion chamber should be 15 times the length of the compression chamber to obtain this condition. The duration of the shock wave from front to trough can be found in terms of  $a_0 t/L_0$  at any distance  $x$  along the tube by measuring the vertical distance from line U to the line  $v_2$ . In similar fashion the duration of the flat top of the shock wave at any point  $x$  is found by a measurement of the vertical distance from line U to line  $b_2$ .

Another method of showing the change in shape of the shock wave while passing through a homogeneous medium is to show the loci of the points relative to the shock front as in the first page of this section. In this case the abscissa is the length or duration of the shock wave while the corresponding ordinate is the distance along the tube or time. In this case the angle of the locus with the ordinate is  $\alpha$  where

$$\tan \alpha = \frac{v}{U} - 1 \quad (\text{VII.17})$$

where  $v$  is the velocity of the component under consideration and  $U$  is the shock wave velocity.

Graph: 7



The critical length of the expansion tube which first allows the rarefaction wave to reach the shock wave may be solved by calculation rather than graphically if we proceed as follows:

After the diaphragm is broken the boundary between cool and hot gas moves down the expansion chamber with a velocity  $u$ , where

$$u = \frac{5a_o(y-1)}{\sqrt{7(1+6y)}} \quad (\text{VII.18})$$

The distance that it has moved at the instant that rarefaction break reaches the end of the compression chamber is

$$d = u T \quad \text{but } T = \frac{L_o}{a_o}$$

so 
$$d = \frac{u L_o}{a_o} \quad (\text{VII.19})$$

The length of the cool gas column at this instant is

$$n = L_o + \frac{u L_o}{a_o} = L_o \left(1 + \frac{u}{a_o}\right) = L_o \left(1 + \frac{5(y-1)}{\sqrt{7(1+6y)}}\right) \quad (\text{VII.20})$$

The time of travel necessary for the rarefaction break to reach the cool gas boundary is

$$T = \frac{n}{b_1 - u} = \frac{n}{c_1} = \frac{L_o}{a_o} \left[ \frac{1 + \frac{5(y-1)}{\sqrt{7(1+6y)}}}{1 - \frac{(y-1)}{\sqrt{7(1+6y)}}} \right] \quad (\text{VII.21})$$

The distance  $x$  from the diaphragm at which the boundary is reached is

$$x = u T + \frac{u L_o}{a_o} = u \left(T + \frac{L_o}{a_o}\right) \quad (\text{VII.22})$$

Substituting from equation (VII.21) we have

$$x = \frac{5 L_o (y-1)}{\sqrt{7(1+6y)}} \left( \frac{2\sqrt{7(1+6y)} + 4(y-1)}{\sqrt{7(1+6y)} - (y-1)} \right) \quad (\text{VII.23})$$

The length of the hot gas column at the time the cool gas boundary has progressed a distance  $x$  down the tube is

$$\delta = T(U - u) \quad \text{where } T = \frac{x}{u}$$

so 
$$\delta = x \left( \frac{U}{b_2} - 1 \right)$$

$$= x \left[ \frac{6+y}{5(y-1)} \right] \quad (\text{VII.24})$$

The additional distance that the rarefaction break must traverse to pass through the hot gas and reach the shock front is

$$x_1 = b_2 T' \quad \text{where} \quad T' = \frac{\delta}{b_2 - U}$$

so 
$$x_1 = \frac{\delta}{1 - \frac{U}{b_2}} \quad \frac{U}{b_2} = \frac{1+6y}{5(y-1) + \sqrt{7y(6+y)}}$$

so 
$$x_1 = x \left\{ \frac{6+y [5(y-1) + \sqrt{7y(6+y)}]}{5(y-1) [\sqrt{7y(6+y)} - (6+y)]} \right\} \quad (\text{VII.25})$$

The total distance from the diaphragm at which the rarefaction break overtakes the shock wave is the sum of these two distances  $x$  and  $x_1$

so 
$$L_e = x + x_1 = x \left[ 1 + \frac{6+y}{5(y-1)} \cdot \frac{\sqrt{7y(6+y)} + 5(y-1)}{\sqrt{7y(6+y)} - (6+y)} \right]$$

We then set  $\phi = \frac{x}{L_e}$

and 
$$\xi = 1 + \frac{(6+y) [\sqrt{7y(6+y)} + 5(y-1)]}{5(y-1) [\sqrt{7y(6+y)} - (6+y)]}$$

so that 
$$\frac{L_e}{L_o} = \xi \phi \quad \text{for the critical length of tube.}$$

The values are tabulated below.

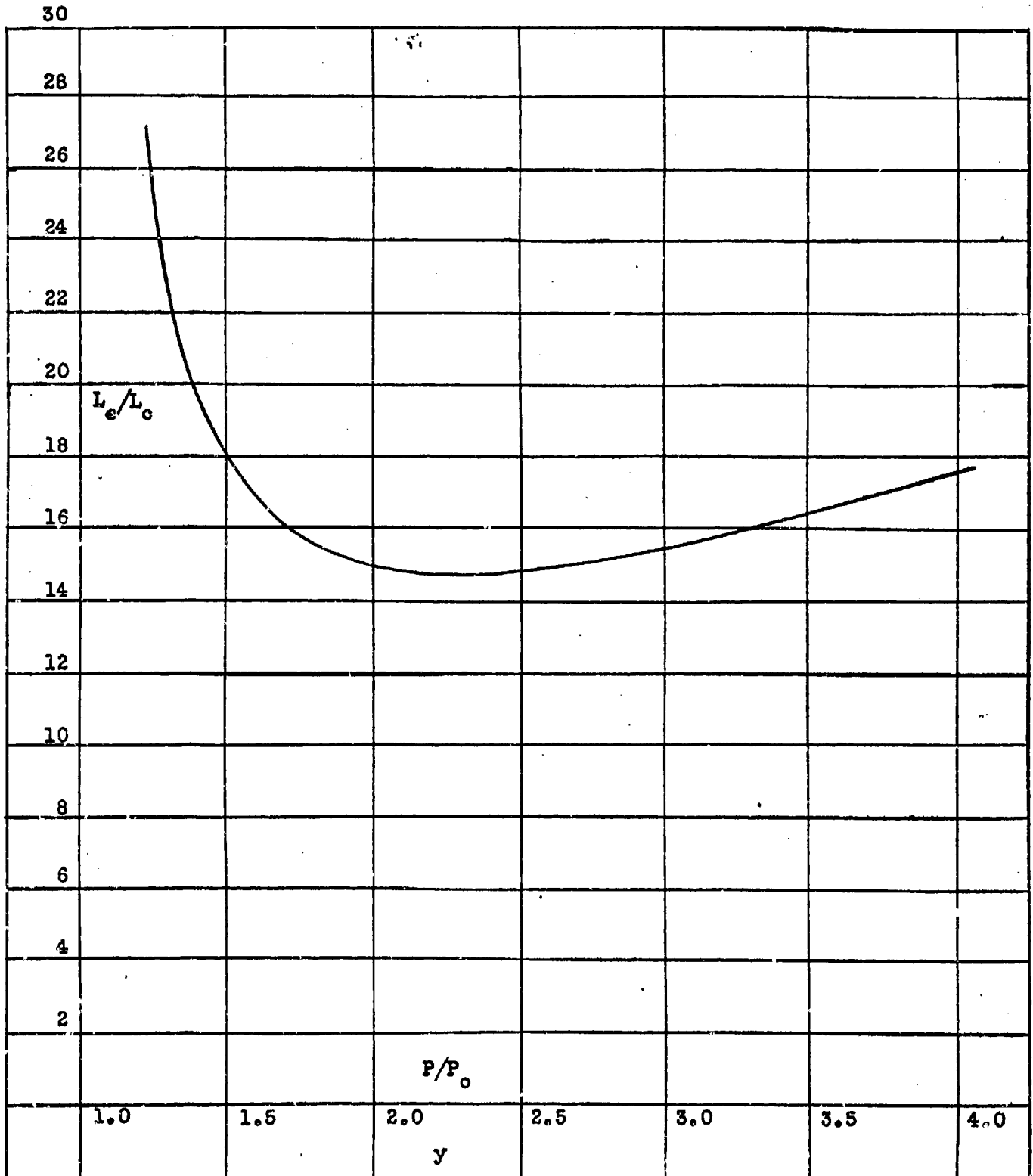
$y$	$P_o/P_o$	$P_{os}$ lb	$\phi$	$\xi$	$\frac{L_e}{L_o}$
1.25	1.558	8.2	0.3565	75.85	27.0
1.50	2.29	18.97	0.712	25.81	18.4
1.75	3.20	32.35	1.064	15.0	15.95
2.00	4.36	49.4	1.418	10.64	15.1
2.50	7.28	92.3	2.12	7.03	14.9
3.00	11.40	153.0	2.82	5.50	15.5
4.00	24.16	340.0	4.255	4.142	17.60

These critical lengths in terms of  $y$  and  $P_{os}$  are plotted on the next pages. (Graphs 8 and 9.)

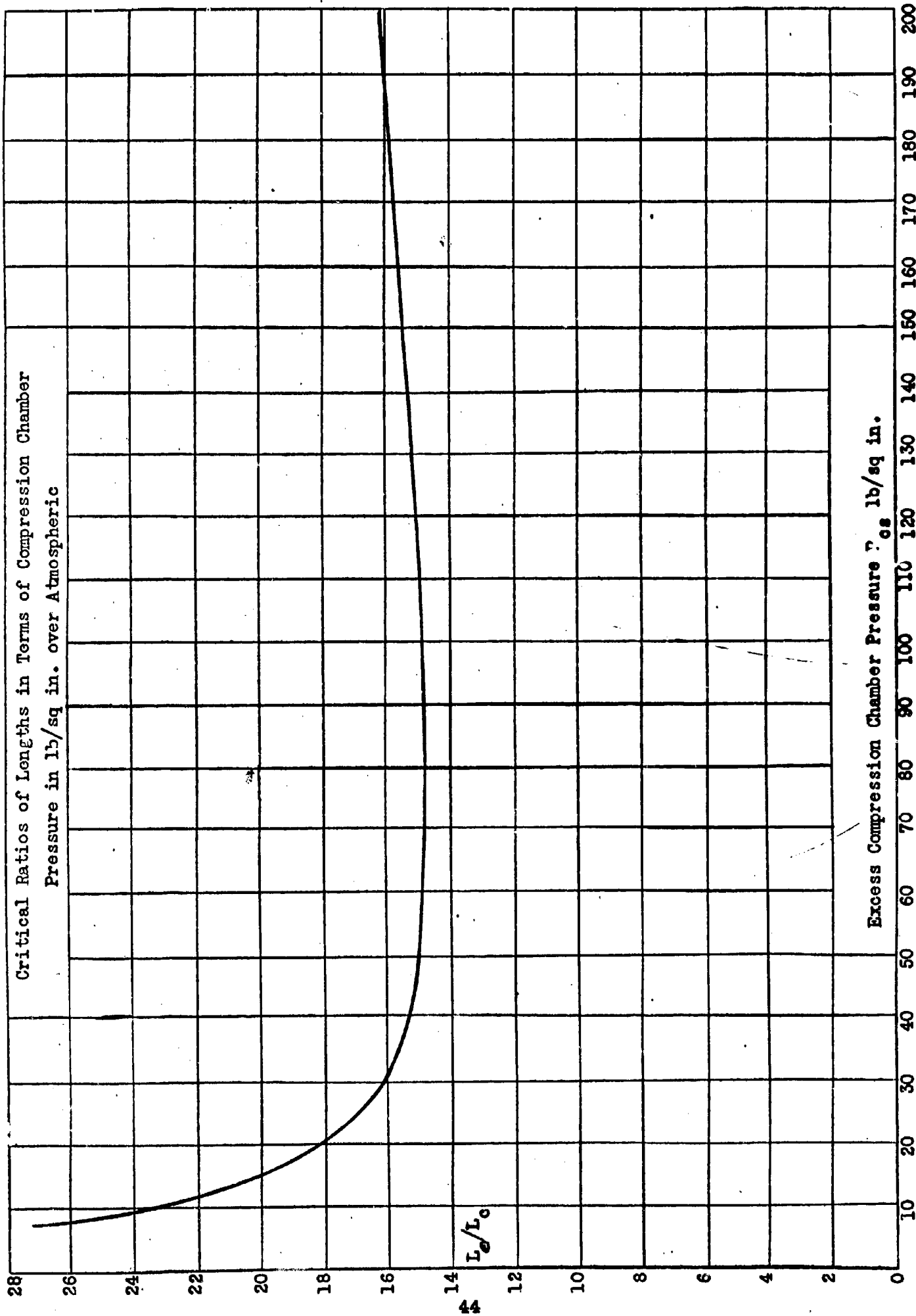
Graph 8

Critical Ratio of Lengths of Expansion Chamber to Length of Compression Chamber as a Function of Shock Strength ( $y$ )

$$y = \frac{P}{P_0}$$



Graph 9



# VIII

## THE IMPULSE IN THE SHOCK WAVE AT THE POINT OF CATCH-UP OF THE RAREFACTION WAVE

A solution of this problem may be attempted by assuming that the energy released from the compression chamber by the expansion of the gases from a pressure  $P_c$  to a pressure  $P_r$ , the pressure in the trough of the reflected rarefaction, is essentially conserved and is the energy available to do work in the gas behind the shock wave. The energy in the shock wave is conceived to be the internal energy of the compressed gases in the shock wave if these gases are expanded adiabatically to the pressure  $P_r$  of the rarefaction wave following the shock wave plus the kinetic energy of the gas particles in the shock wave. The assumption of conservation of energy will be examined later to determine the error introduced by it.

If the concept of the energy in the shock wave outlined above is adopted it will then be permissible to assume a relationship between the energy in the shock wave and the impulse associated with it of the following form

$$I = \frac{kE}{U} \quad (\text{VIII.1})$$

where  $E$  = energy of the shock wave  
 $I$  = impulse of the shock wave as measured by a gauge side on  
 $U$  = velocity of the shock wave  
 and  $k$  = a dimensionless factor to be determined later. It is a function of the shock strength and the shape of the pressure-time curve of the shock wave.

Equation (VIII.1) will be used to calculate the impulse of the shock wave when we have evaluated  $k$  and  $E$ , the energy in the shock.

The work done by a unit mass of gas in adiabatically expanding from a pressure  $P_c$  to a pressure  $P_r$  is

$$e_c - e_r = \frac{1}{\gamma - 1} \left[ \frac{P_c}{\rho_c} - \frac{P_r}{\rho_r} \right] = \frac{1}{\gamma - 1} \frac{P_c}{\rho_c} \left[ 1 - \frac{P_r}{P_c} \frac{\rho_c}{\rho_r} \right] \quad (\text{VIII.2})$$

$$\text{but } \frac{\rho_c}{\rho_r} = \left( \frac{P_c}{P_r} \right)^{1/\gamma}$$

$$\text{so } e_c - e_r = \frac{1}{\gamma - 1} \frac{P_c}{\rho_c} \left[ 1 - \left( \frac{P_c}{P_r} \right)^{\frac{1-\gamma}{\gamma}} \right] \quad \text{energy per unit mass of gas.} \quad (\text{VIII.2a})$$

The available energy per unit volume of gas at a density  $\rho_c$  in a tube of length  $L_c$  and unit area is

$$e = \rho_c L_c (e_c - e_r)$$

$$\text{so } e = \frac{P_c L_c}{\gamma - 1} \left[ 1 - \left( \frac{P_c}{P_r} \right)^{\frac{1-\gamma}{\gamma}} \right] \quad \text{and if } \gamma = 1.4$$

$$e = \frac{5}{2} P_c L_c \left[ 1 - \left( \frac{P_c}{P_r} \right)^{-2/7} \right] \quad (\text{VIII.3})$$

This is the energy available for adiabatic expansion in a cylinder of gas of unit cross section, of length  $L_c$ , at a pressure  $P_c$  and expanded to a pressure  $P_r$ .

If  $P$  is the peak pressure in the shock wave and  $P_o$  is the initial pressure in the compression and expansion chamber we then define the ratio

$$P/P_o = y \quad \text{and} \quad P_c/P_o = \omega$$

If we consider the initial formation of the wave to be adiabatic we can get an explicit expression for  $y$  in terms of  $\omega$  which is:

$$y^{1/7} = \frac{2\omega^{1/7}}{\omega^{1/7} + 1} \quad \text{equation (VI.5a)} \quad (\text{VIII.4})$$

This assumption which is made for computational convenience results in an error which may be determined for any range of shock pressures from the table at the end of section VI. For the range of pressures considered here it may be responsible for a maximum error of about 2 percent at the highest shock pressure.

From the theory of reflection of adiabatic waves we find the ratio

$$\left( \frac{P_r}{P_o} \right)^{1/7} = 2 y^{1/7} - \omega^{1/7} \quad \text{equation (II.8b)} \quad (\text{VIII.5})$$

consequently in terms of  $\omega$

$$\left( \frac{P_r}{P_o} \right)^{1/7} = \frac{\omega^{1/7} (3 - \omega^{1/7})}{\omega^{1/7} + 1} \quad (\text{VIII.6})$$

$$\text{but } \left( \frac{P_c}{P_r} \right)^{1/7} = \left( \frac{P_c}{P_o} \right)^{1/7} \left( \frac{P_o}{P_r} \right)^{1/7}$$

$$\text{so } \left( \frac{P_c}{P_r} \right)^{-2/7} = \left( \frac{3 - \omega^{1/7}}{\omega^{1/7} + 1} \right)^2 \quad (\text{VIII.7})$$

$$\text{then } E = \frac{5}{2} P_c L_c \left[ \frac{8(\omega^{1/7} - 1)}{(\omega^{1/7} + 1)^2} \right] \quad (\text{VIII.8})$$

which is also equal to  $E = 20 P_o L_o \left[ \frac{\omega(\omega^{1/7} - 1)}{(\omega^{1/7} + 1)^2} \right]$  (VIII.8a)

This equation (VIII.8a) then gives the value in terms of  $\omega$  of the available energy in a tube of unit cross section and length  $L_o$  when expanded to a pressure  $P$  which is determined by the reflection of the rarefaction wave produced by a bursting diaphragm.

If a shock wave of pressure  $P$  is traveling into undisturbed air of pressure  $P_o$  the velocity of the shock wave is  $U$ , where  $U$  is given by the equation

$$U = a \sqrt{\frac{1 + 6y}{7}} \quad (\text{VIII.9})$$

Again using the explicit expression for  $y$  in terms of  $\omega$  given above for the adiabatic case we have

$$U = \frac{a}{\sqrt{7}} \left[ 1 + \frac{768 \omega}{(\omega^{1/7} + 1)^7} \right]^{1/2} \quad \begin{array}{l} \text{velocity of shock} \\ \text{wave in terms of} \\ \omega = P/P_o \end{array} \quad (\text{VIII.10})$$

We now have expressions for two of the factors involved in equation (VIII.1) and it remains to evaluate the factor  $k$  in order to arrive at an answer. One might infer from an analogy with the mechanical case where  $k = 2$  that the factor might lie in the neighborhood of this value. The concept of stored energy in the shock wave coupled with some reasonable assumptions as to the shape of the pressure-time curve of the shock wave enables us to make a reasonably accurate determination of this factor which does turn out to be of the order of magnitude of 2.

We proceed to evaluate  $k$  in the following manner.

From equation (VIII.1) we see that

$$k = \frac{IU}{E} \quad \text{for any blast wave.} \quad (\text{VIII.11})$$

The energy in a shock wave consists partly of potential and partly of kinetic energy, the proportions of which change with the amplitude of the shock. The potential energy is considered to be the available pressure energy of the gas in the shock wave, while the kinetic energy is the energy of motion of the particles in the shock wave.

The available pressure energy of a slice of gas somewhere behind the shock front is

$$\Delta E_1 = \frac{1}{\gamma-1} \left( \frac{P}{\rho} - \frac{P_o}{\rho_o} \right) = \frac{1}{\gamma-1} \frac{P_o}{\rho_o} \left( \frac{P}{P_o} - \frac{\rho_o}{\rho} - 1 \right) \quad (\text{VIII.12})$$

but  $P/P_0 = y$  and  $\rho/\rho_0 = x$

so

$$\Delta E_1 = \frac{1}{\gamma-1} \frac{P_0}{\rho_0} \left( \frac{y}{x} - 1 \right) \quad \begin{array}{l} \text{Energy per unit mass of gas} \\ \text{if the gas were expanded} \\ \text{adiabatically to a pressure} \\ P_0 \text{ and density } \rho_0. \end{array} \quad (\text{VIII.12a})$$

The energy per unit volume is  $\rho \Delta E_1$  and is

$$\Delta E_p = \frac{1}{\gamma-1} P_0 (y - x) \quad (\text{VIII.13})$$

and if  $\gamma = 1.4$  (air)

$$\text{then } \Delta E_p = \frac{5}{2} P_0 (y - x) \quad (\text{VIII.13a})$$

From equation (III.5) section III, we find that the ratio of densities  $x$  in a shock wave is

$$x = \frac{1 + 6y}{6 + y}$$

so

$$\Delta E_p = \frac{5}{2} P_0 \left( \frac{y^2 - 1}{6 + y} \right) \quad \begin{array}{l} \text{Potential energy in a slice of} \\ \text{gas of unit cross section and} \\ \text{unit length at pressure } P \text{ and} \\ \text{density } \rho. \end{array} \quad (\text{VIII.13b})$$

The total potential energy in the blast wave is then

$$E_p = \frac{5}{2} P_0 \int_0^{\bar{x}} \frac{y^2 - 1}{6 + y} dx \quad (\text{VIII.14})$$

The kinetic energy per slice of length  $dx$  and unit area is

$$\Delta E_2 = \frac{1}{2} \rho u^2 dx \quad \begin{array}{l} \text{where } u \text{ is the particle velocity} \\ \text{and } \rho \text{ is the density} \end{array} \quad (\text{VIII.15})$$

$$= \frac{1}{2} \rho_0 \left( \frac{\rho}{\rho_0} \right) u^2 dx \quad (\text{VIII.15a})$$

$$\text{but } u^2 = \frac{25 a^2 (y-1)^2}{7(1+6y)} \quad \text{and} \quad \frac{\rho}{\rho_0} = \frac{1+6y}{6+y}$$

$$\text{so } \Delta E_2 = \frac{1}{2} \rho_0 \frac{25 a^2}{7} \frac{(y-1)^2}{6+y} dx \quad (\text{VIII.15b})$$

and the total kinetic energy in the wave is then

$$E_k = \frac{25 a^2 \rho_o}{14} \int_0^x \frac{(y-1)^2 dx}{6+y} \quad (\text{VIII.16})$$

but  $a^2 = \frac{7}{5} \frac{P_o}{\rho_o}$

so 
$$E_k = \frac{5}{2} P_o \int_0^x \frac{(y-1)^2 dx}{6+y} \quad (\text{VIII.16a})$$

The total available energy in the shock wave is then the sum of the kinetic and potential energies which is

$$\begin{aligned} E_t &= \frac{5}{2} P_o \int_0^x \frac{y^2 - 1 + (y-1)^2 dx}{6+y} \\ &= 5 P_o \int_0^x \frac{y(y-1) dx}{6+y} \quad \begin{array}{l} \text{total energy in the} \\ \text{shock wave} \end{array} \quad (\text{VIII.17}) \end{aligned}$$

Now if the shock wave changes in duration slowly enough while passing over its own length we may put

$$dx = U dt \quad (\text{VIII.18})$$

so that these equations may be written as

$$E_p = \frac{5}{2} P_o U \int_0^t \frac{y^2 - 1}{6+y} dt \quad (\text{VIII.19})$$

$$E_k = \frac{5}{2} P_o U \int_0^t \frac{(y-1)^2}{6+y} dt \quad (\text{VIII.20})$$

$$E_t = 5 P_o U \int_0^t \frac{y(y-1)}{6+y} dt \quad (\text{VIII.21})$$

We may substitute  $z = y - 1$  as a variable and make an assumption as to the rate of change of  $z$  with time. Oscillograms of the pressure-time variation show that the wave shape approaches triangular form at the lower pressures while at the higher pressures it approaches an exponential form.

Assume that the wave shape is exponential and that

$$z = \bar{z} e^{-\alpha t} \quad \text{then} \quad dt = -\frac{1}{\alpha} \frac{dz}{z}$$

when  $z = 0$ ,  $t = \infty$ ,  $z = \bar{z}$ ,  $t = 0$

so that equation (VIII.21) may be written after substituting for  $y$  and  $dt$

$$E_t = \frac{5 P_o U}{\alpha} \int_0^{\bar{z}} \frac{(z+1) dz}{7+z} \quad (\text{VIII.22})$$

$$\begin{aligned} E_t &= \frac{5 P_o U}{\alpha} \left[ \int_0^{\bar{z}} \frac{z dz}{7+z} + \int_0^{\bar{z}} \frac{dz}{7+z} \right] \\ &= \frac{5 P_o U}{\alpha} \left[ 7 + z - 7 \log (7+z) \right]_0^{\bar{z}} + \left[ \log (7+z) \right]_0^{\bar{z}} \\ &= \frac{5 P_o U}{\alpha} \left[ \bar{z} - 6 \log \left( 1 + \frac{\bar{z}}{7} \right) \right] \end{aligned} \quad (\text{VIII.23})$$

The impulse measured side on in a blast wave is defined as

$$I = \int_0^t P_s dt = P_o \int_0^t (y+1) dt = P_o \int_0^t z dt \quad (\text{VIII.24})$$

but  $dt = -\frac{1}{\alpha} \frac{dz}{z}$

$$\text{so} \quad I = \frac{P_o}{\alpha} \int_0^{\bar{z}} dz = \frac{P_o \bar{z}}{\alpha} \quad (\text{VIII.25})$$

$$\text{Then} \quad IU = \frac{P_o U \bar{z}}{\alpha}$$

$$\text{and} \quad k = \frac{IU}{E}$$

therefore the value of the factor  $k$  may be written as

$$k = \frac{z}{5 \left[ \bar{z} - 6 \log \left( 1 + \frac{\bar{z}}{7} \right) \right]} \quad (\text{VIII.26})$$

The term  $\log \left( 1 + \frac{\bar{z}}{7} \right)$  may be expanded in a series to give the approximate formula

$$k = \frac{7}{5 \left( 1 + \frac{3\bar{z}}{7} - \frac{2\bar{z}^2}{49} + \frac{3\bar{z}^3}{686} \dots \right)} \quad (\text{VIII.26a})$$

which has the value  $7/5$  when  $\bar{z} \rightarrow 0$ . This is of the order of magnitude of the mechanical factor 2 as mentioned before.

If the explicit relationship  $y = \frac{128 \omega}{(\omega^{1/7} + 1)^7}$  is used the factor  $k$  may be written

$$k = \frac{128 \omega - (\omega^{1/7} + 1)^7}{5 \left[ 128 \omega - (\omega^{1/7} + 1)^7 \left\{ 1 + 6 \log_{10} \left( \frac{6}{7} + \frac{128 \omega}{7(\omega^{1/7} + 1)^7} \right) \right\} \right]} \quad (\text{VIII.26b})$$

Recapitulating the formulas we have

$$I = \frac{KE}{U} \quad (\text{VIII.11})$$

$$k = \frac{128 \omega - (\omega^{1/7} + 1)^7}{5 \left[ 128 \omega - (\omega^{1/7} + 1)^7 \left\{ 1 + 6 \log_{10} \left( \frac{6}{7} + \frac{128 \omega}{7(\omega^{1/7} + 1)^7} \right) \right\} \right]} \quad (\text{VIII.26b})$$

$$E = 20 P_o L_o \left[ \frac{\omega(\omega^{1/7} - 1)}{(\omega^{1/7} + 1)^2} \right] \quad (\text{VIII.8a})$$

$$U = \frac{a_o}{\sqrt{7}} \left[ 1 + \frac{768 \omega}{(\omega^{1/7} + 1)^7} \right]^{1/2} \quad (\text{VIII.10})$$

Combining these factors we then have the expression for the impulse in the shock wave in terms of the dimensionless quantity  $I a_o / P_o L_o$ .

This equation will hold at the point of catch up of the rarefaction wave with the shock front if the energy lost by the shock wave in passing down the critical length of tube is ignored. This will result in an error of less than 4 percent in the calculated value of the impulse. We have then

$$\frac{I a_o}{P_o L_o} = 10.58 \frac{[\omega(\omega^{1/7} - 1)]}{[\omega^{1/7} + 1]^2} \quad (\text{VIII.27})$$

$$\frac{[128 \omega - (\omega^{1/7} + 1)^7]}{[128 \omega - (\omega^{1/7} + 1)^7 \left\{ 1 + 6 \log_{10} \left( \frac{6}{7} + \frac{128 \omega}{7(\omega^{1/7} + 1)^7} \right) \right\}]} \left[ 1 + \frac{768 \omega}{(\omega^{1/7} + 1)^7} \right]^{1/2}$$

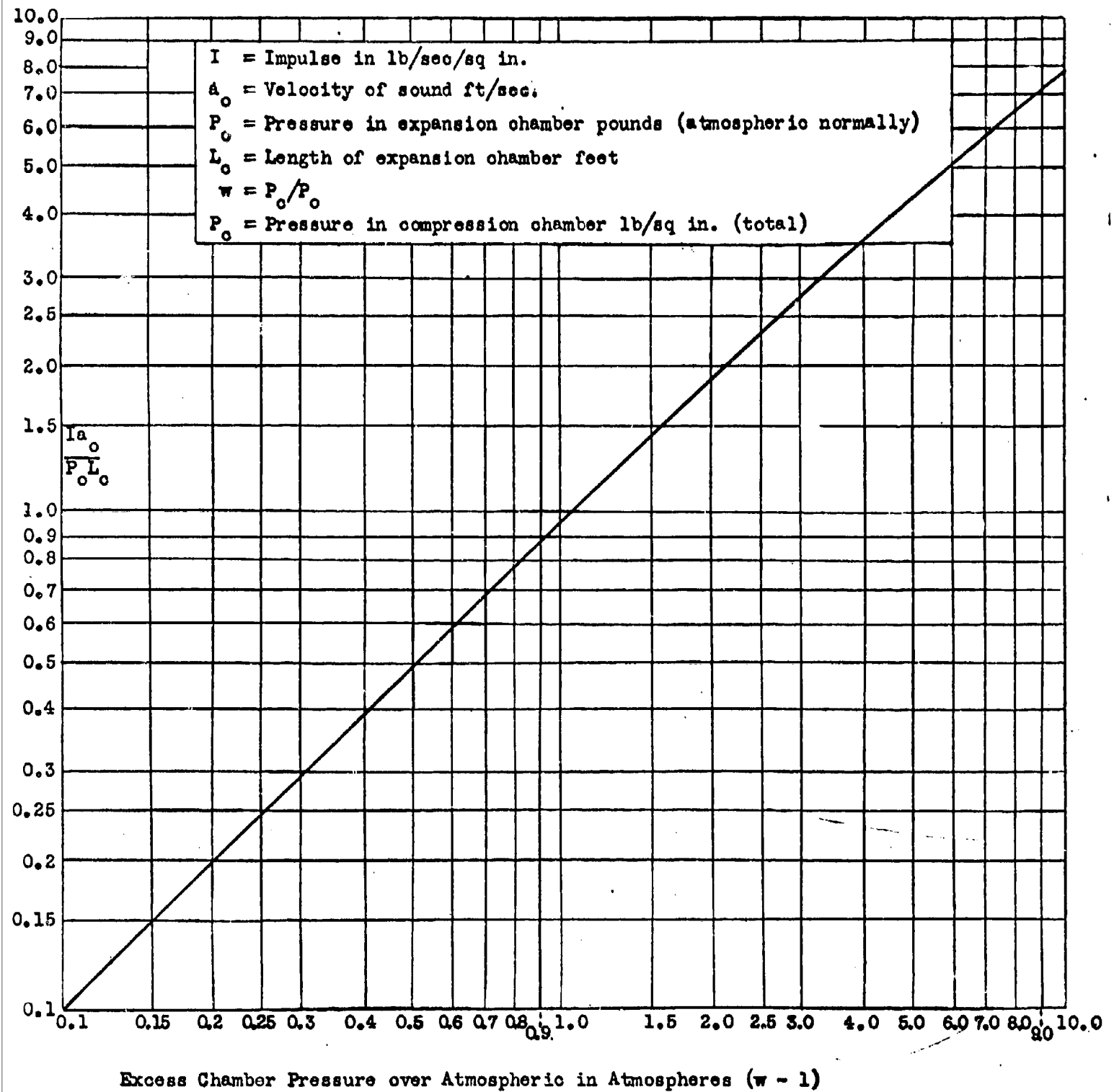
This relation is plotted in terms of  $(\omega - 1)$ , Graph 10.

## IX. THE CHANGE OF ENERGY AS THE SHOCK WAVE PROGRESSES ALONG THE EXPANSION CHAMBER

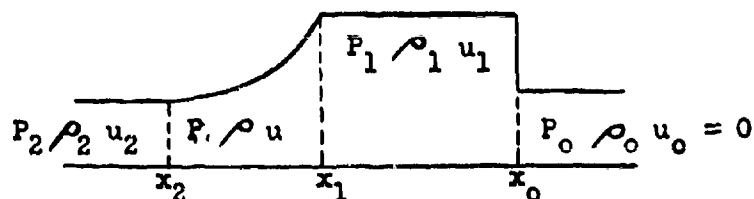
When a shock wave passes over a mass of gas it changes the entropy of the gas and leaves it in a different energy state in general than it was originally. This change in energy can be found by differentiating with respect to time the equations which represent the energy of the shock wave.

Graph 10

Impulse in Shock Wave as a Function of Excess Pressure in Compression Chamber



If one considers a shock wave in which the pressures, densities, and particle velocities have values indicated by the subscripts on the diagram as follows,



the coordinates at any instant of time of the boundaries between these regions have the values  $x_0$ ,  $x_1$  and  $x_2$ . The total energy in the shock wave then is given by the integral of the sum of the kinetic energies of the particles and the increase of internal energy of the gas. This is given below as

$$E = \int_{x_1}^{x_0} \rho_1 \left( \frac{1}{2} u_1^2 + E_1 - E_0 \right) dx + \int_{x_2}^{x_1} \rho \left( \frac{1}{2} u^2 + E - E_0 \right) dx \quad (\text{IX.1})$$

The change of energy with respect to time is the derivative of this expression with respect to time and is

$$\begin{aligned} \frac{\partial E}{\partial t} = & \rho_1 \left( \frac{1}{2} u_1^2 + E_1 - E_0 \right) \frac{\partial x_0}{\partial t} - \rho_2 \left( \frac{1}{2} u_2^2 + E_2 - E_0 \right) \frac{\partial x_2}{\partial t} \\ & + \int_{x_2}^{x_1} \frac{\partial}{\partial t} \rho \left( \frac{1}{2} u^2 + E - E_0 \right) dx \end{aligned} \quad (\text{IX.2})$$

Now  $\frac{\partial x_0}{\partial t} = U$  the velocity of the shock wave

and

$$\frac{\partial x_2}{\partial t} = a_2 + u_2 \quad \text{where } a_2 \text{ is the velocity of sound in the trough and } u_2 \text{ is the particle velocity in the trough.} \quad (\text{IX.3})$$

So the rate of energy change is

$$\begin{aligned} \frac{\partial E}{\partial t} = & \rho_1 U \left( \frac{1}{2} u_1^2 + E_1 - E_0 \right) - \rho_2 (a_2 + u_2) \left( \frac{1}{2} u_2^2 + E_2 - E_0 \right) \\ & + \int_{x_2}^{x_1} \frac{\partial}{\partial t} \rho \left( \frac{1}{2} u^2 + E - E_0 \right) dx \end{aligned} \quad (\text{IX.4})$$

The last term in the integral sign is equal to

$$\int_{x_2}^{x_1} \frac{\partial}{\partial t} \left( \frac{1}{2} \rho u^2 + \frac{5}{2} P - \frac{5}{2} \frac{P_0}{\rho_0} \rho \right) dx =$$

$$\int_{x_2}^x \left( \rho u \frac{\partial u}{\partial t} + \frac{1}{2} u^2 \frac{\partial \rho}{\partial t} + \frac{5}{2} \frac{\partial P}{\partial t} + \frac{5}{2} \frac{P_0}{\rho_0} \frac{\partial \rho}{\partial t} \right) dx \quad (\text{IX.4a})$$

but from the equation of motion and continuity we have

$$\frac{\partial \rho}{\partial t} = -\rho \frac{\partial u}{\partial x} - u \frac{\partial \rho}{\partial x} \quad (\text{IX.5})$$

$$\frac{\partial P}{\partial t} = -u \frac{\partial P}{\partial x} - \frac{1}{\rho} \frac{\partial P}{\partial x} \quad (\text{IX.6})$$

Substituting these values the integral becomes

$$\int_{x_2}^{x_1} \left( -\frac{3}{2} \rho u^2 \frac{\partial u}{\partial x} - \frac{1}{2} u^3 \frac{\partial \rho}{\partial x} + \frac{5}{2} \frac{P_0}{\rho_0} \rho \frac{\partial u}{\partial x} + \frac{5}{2} \frac{P_0}{\rho_0} u \frac{\partial \rho}{\partial x} \right.$$

$$\left. - \frac{7}{2} \frac{\partial P}{\partial \rho} u \frac{\partial \rho}{\partial x} - \frac{5}{2} \frac{\partial P}{\partial \rho} \rho \frac{\partial u}{\partial x} \right) dx = \int_{x_2}^{x_1} -\frac{\partial}{\partial x} \left( \frac{1}{2} \rho u^3 \right)$$

$$+ \frac{\partial}{\partial x} \left( \frac{5}{2} \frac{P_0}{\rho_0} \rho u \right) - \frac{7}{2} \left( u \frac{\partial P}{\partial x} - \frac{5}{7} \frac{\partial P}{\partial \rho} \rho \frac{\partial u}{\partial x} \right) \quad (\text{IX.7})$$

$$\text{but } \frac{5}{7} \frac{\partial P}{\partial \rho} \rho = \frac{5}{7} \cdot \frac{7}{5} \cdot \frac{P}{\rho} \cdot \rho = P \quad (\text{IX.8})$$

So then performing the integration and substituting in the limits we find that it is equal to

$$-\frac{1}{2} \rho_1 u_1^3 + \frac{5}{2} \frac{P_0}{\rho_0} \rho_1 u_1 - \frac{7}{2} P_1 u_1 + \frac{1}{2} \rho_2 u_2^3 - \frac{5}{2} \frac{P_0}{\rho_0} \rho_2 u_2$$

$$+ \frac{7}{2} P_2 u_2 \quad (\text{IX.9})$$

Then adding all terms we have

$$\frac{\partial E}{\partial t} = \rho_1 u \left( \frac{1}{2} u_1^2 + E_1 - E_0 \right) - \rho_2 u \left( \frac{1}{2} u_2^2 + E_2 - E_0 \right) - \rho_1 u_1 \left( \frac{1}{2} u_1^2 + E_1 - E_0 \right)$$

$$- P_1 u_1 + P_2 u_2 \quad (\text{IX.10})$$

and since  $\rho_1(U - u_1) = \rho_0 U$  by the equation for conservation of mass, then

$$\frac{\partial E}{\partial t} = \rho_0 U \left( \frac{1}{2} u_1^2 + E_1 - E_0 \right) - P_1 u_1 - \rho_2 a_2 \left( \frac{1}{2} u_2^2 + E_2 - E_0 \right) + P_2 u_2 \quad (\text{IX.11})$$

However as a consequence of the conservation of energy as defined in the Rankine-Hugoniot equations the first two terms cancel so we are left with the expression for energy change as follows:

$$\frac{\partial E}{\partial t} = - \rho_2 a_2 \left( \frac{1}{2} u_2^2 + E_2 - E_0 \right) + P_2 u_2 \quad \text{where } u_2 \text{ must be given the proper vector sign.} \quad (\text{IX.12})$$

This equation expresses the idea that the energy change as a function of time is the difference between the total energies of the gas as it enters and leaves the shock wave plus the work done on or by the gas after it leaves the shock wave depending on whether  $u_2$  has a positive or negative sign.

Let us consider the shock wave after the rarefaction trough has passed the gas boundary. The reason for doing so is that until this stage is reached the gas in the compression chamber is still giving up energy to the shock wave but after this stage the shock wave starts to dissipate and to feed back a small amount of its energy into the compression chamber in the form of a compression wave.

After this point  $u_2 = u - u_r$

$$\text{where } u = 5c \frac{y-1}{\sqrt{7y(6+y)}} \quad \text{and } u_r = 5c \left[ 1 - \left( \frac{P_2}{P} \right)^{1/7} \right] \quad (\text{IX.13})$$

$$\text{so } u_2 = 5c \left[ \frac{y-1}{\sqrt{7y(6+y)}} - 1 + \left( \frac{P_2}{P} \right)^{1/7} \right] \quad (\text{IX.14})$$

$$c = a_0 \sqrt{\frac{y(6+y)}{1+6y}} \quad \text{where } y = \frac{P}{P_0} \quad (\text{IX.15})$$

Now from the theory of reflection and transmission of rarefaction waves as given in section II, we have

$$\left( \frac{P_2}{P} \right)^{1/7} = \frac{2a_1}{c + a_1} \left( \frac{P_{r1}}{P} \right)^{1/7} + \frac{c - a_1}{c + a_1} \quad (\text{IX.16})$$

where  $c$  = speed of sound in the shock wave

$a_1$  = speed of sound in gas that has been cooled by expansion from pressure  $P_0$  to  $P_{r1}$ .

From section II we have

$$\left(\frac{P_{r1}}{P}\right)^{1/7} = 2 - \left(\frac{P_0}{P}\right)^{1/7} \quad (\text{IX.17})$$

but  $\left(\frac{P_0}{P}\right)^{1/7} = \frac{1}{1 - \frac{(y-1)}{\sqrt{7(1+6y)}}}$  (IX.18)

so  $\left(\frac{P_{r1}}{P}\right)^{1/7} = \frac{\sqrt{7(1+6y)} - 2(y-1)}{\sqrt{7(1+6y)} - (y-1)}$  (IX.19)

and  $a_1 = a_0 \left(\frac{P_{r1}}{P_0}\right)^{1/7} = a_0 \left(\frac{P_{r1}}{P}\right)^{1/7} \left(\frac{P}{P_0}\right)^{1/7} =$   
 $a_0 \left[ 1 - \frac{2(y-1)}{\sqrt{7(1+6y)}} \right]$  (IX.20)

The ratio of densities  $\rho_2/\rho_0$  can be found by considering the gas to be compressed according to the Rankine-Hugoniot equation of state and to expand according to the adiabatic equation of state so that

$$\frac{\rho}{\rho_0} = \frac{1+6y}{6+y} \quad \text{and} \quad \frac{\rho_2}{\rho} = \left(\frac{P_2}{P}\right)^{5/7}$$

then  $\frac{\rho_2}{\rho_0} \cdot \frac{\rho_0}{\rho} = \left(\frac{P_2}{P_0}\right)^{5/7} \cdot \left(\frac{P_0}{P}\right)^{5/7}$  (IX.21)

then  $\frac{\rho_0}{\rho_2} = y^{5/7} \cdot \frac{\rho_0}{\rho} \cdot \left(\frac{P_2}{P_0}\right)^{-5/7} = y^{5/7} \cdot \frac{(6+y)}{(1+6y)} \cdot \left(\frac{P_2}{P_0}\right)^{-5/7}$  (IX.22)

then  $\rho_2 = \left(\frac{P_2}{P_0}\right)^{5/7} \left[ \frac{1+6y}{y^{5/7}(6+y)} \right] \rho_0 = \rho_0 \left(\frac{P_2}{P_0}\right)^{5/7} \cdot \frac{1+6y}{6+y}$  (IX.23)

The change of internal energy  $E_2 - E_0$  is equal to

$$E_2 - E_0 = \frac{5}{2} \left( \frac{P_2}{\rho_2} - \frac{P_0}{\rho_0} \right) = \frac{5}{2} \frac{P_0}{\rho_0} \left( \frac{P_2}{P_0} \right)^{5/7} \cdot \frac{\rho_0}{\rho_2} - 1 \quad (\text{IX.24})$$

Substituting for  $\rho_0/\rho_2$  we have

$$E_2 - E_0 = \frac{5}{2} \frac{P_0}{\rho_0} \left[ \frac{y^{5/7}(6+y)}{1+6y} \left(\frac{P_2}{P_0}\right)^{2/7} - 1 \right] \quad (\text{IX.25})$$

and

$$a_2 = a_0 \left(\frac{P_2}{P_0}\right)^{1/7} = a_0 \sqrt{\frac{y(6+y)}{1+6y}} \left(\frac{P_2}{P_0}\right)^{1/7} \quad (\text{IX.26})$$

Example  $y = 2$

$$a = a_0 \sqrt{\frac{y(6+y)}{1+6y}} = a_0 \sqrt{\frac{16}{13}} = 1.11 a_0 \quad \frac{P_2}{P_0} = 0.977$$

$$a_1 = 0.790 a_0$$

$$\left(\frac{P_2}{P_0}\right)^{1/7} = 0.902$$

$$\left(\frac{P_2}{P_0}\right)^{1/7} = 0.9965$$

$$u_2 = -0.0194 a_0$$

$$\frac{1}{2} u_2^2 = 0.000263 \frac{P_0}{\rho_0}$$

$$\rho_2 = 0.978 \rho_0$$

$$E_2 - E_0 = 0.0025 \frac{P_0}{\rho_0}$$

$$a_2 = 1.00 a_0$$

then

$$\frac{\partial E}{\partial t} = -P_0 a_0 \left[ 0.978 \times 1.00 (0.000263 + 0.0025) - (0.977 \times 0.0194) \right] = P_0 a_0 (0.0165)$$

The total dissipation  $E_d = \frac{\partial E}{\partial t} t = 0.0165 P_0 a_0 t$

From the diagram of section VII we find that the transit time of the rarefaction trough from the gas boundary to the point of catch up is

$$11 \frac{L_c}{a_0} \text{ so that } a_0 t = 11 L_c.$$

Therefore the total dissipation  $E_d$  is  $0.181 P_0 L_c$ .

The energy originally in the wave is

$$E = 10 P_o L_o \left[ \frac{y(y^{1/7} - 1)}{(2 - y^{1/7})^6} \right] = 4.03 P_o L_o \quad (\text{IX.27})$$

so that the ratio of the energy dissipated to the total original energy is

$$\frac{E_d}{E} = \frac{0.181}{4.03} = 0.045 \cong 5\%$$

This gives a measure of the accuracy of the calculations of impulse which are based among other things upon the assumption of conservation of energy of the shock wave. Presumably then at the pressure level  $y = 2$  the error in the calculations should be of the order of 5 percent.

#### X. AN EXPERIMENTAL DETERMINATION OF THE POINT OF CATCH UP OF THE RAREFACTION WAVE AND THE IMPULSE OF THE SHOCK WAVE IN A TUBE

##### ABSTRACT

Experiments are reported here which show that:

(A) The impulse in the shock wave in a tube at the critical distance is very nearly proportional to the excess chamber pressure and can be calculated by considerations of the energy in the shock wave.

(B) The length of expansion tube necessary to allow the rarefaction wave to catch the shock front and to produce a peaked shock wave can be found from a knowledge of the various wave velocities and is of the order of magnitude of fifteen times the compression chamber length at the higher shock pressures.

Note: The complete paper on this subject will be found in AES  
7 February 1945.

## APPENDIX A

### A SIMPLE DERIVATION OF THE EQUATION FOR THE TOTAL ENERGY IN A SHOCK WAVE

Assume a tube of unit cross section open to the atmospheric pressure  $P_0$  at one end and closed at the other end by a weightless piston. The air behind the piston is evacuated to avoid any complications due to the formation of rarefaction waves so that the static pressure on the piston is  $P_0$ . Now let the piston be acted upon by a suddenly applied uniform force  $P$  which causes it to move with a uniform velocity  $u$  until the piston has moved a distance  $x_0$ . The total work done by the piston is then  $Px_0$ .

The motion of the piston will produce a region of compressed gas ahead of the piston whose pressure is  $P$  and whose boundary will be a shock front of velocity  $U$ . The velocity of the piston will be  $u$ , the velocity of the gas particles behind a shock front of velocity  $U$  and pressure  $P$ . The shock front will then move a distance  $Ut$  while the piston moves a distance  $ut$ . The length of the shock wave at the time  $t$  will be  $(U - u)t$ . Let  $t = \frac{x_0}{u}$  then  $x_1$  the length of the shock wave at the time the piston has moved a distance  $x_0$  will be

$$x_1 = \frac{U - u}{u} x_0 \quad \text{and} \quad \frac{x_1}{x_0} = \frac{U}{u} - 1 \quad (\text{A.1})$$

The energy in the shock wave will then be equal to the energy per unit volume times the volume of the shock wave.

Since the tube is of unit cross section then this is

$$\left(\frac{dE}{dx}\right) \cdot x_1 = E_t \quad \text{the total energy in the shock wave} \quad (\text{A.2})$$

This in turn must be equal to the work done on the piston which is  $Px_0$ .

$$\therefore \frac{dE}{dx} = P \frac{x_0}{x_1} = P_0 y \frac{x_0}{x_1} \quad \text{where} \quad y = \frac{P}{P_0} \quad (\text{A.3})$$

But from the shock wave equations derived from the condition of the Rankine-Hugoniot equations we have

$$\frac{U}{u} = \frac{1 + 6y}{5(y - 1)} \quad (\gamma = 1.4) \quad (\text{A.4})$$

$$\text{and} \quad \frac{U}{u} - 1 = \frac{6 + y}{5(y - 1)} \quad (\text{A.5})$$

$$\frac{x_0}{x_1} = \frac{5(y - 1)}{6 + y} \quad (\text{A.6})$$

Consequently for a flat top shock wave

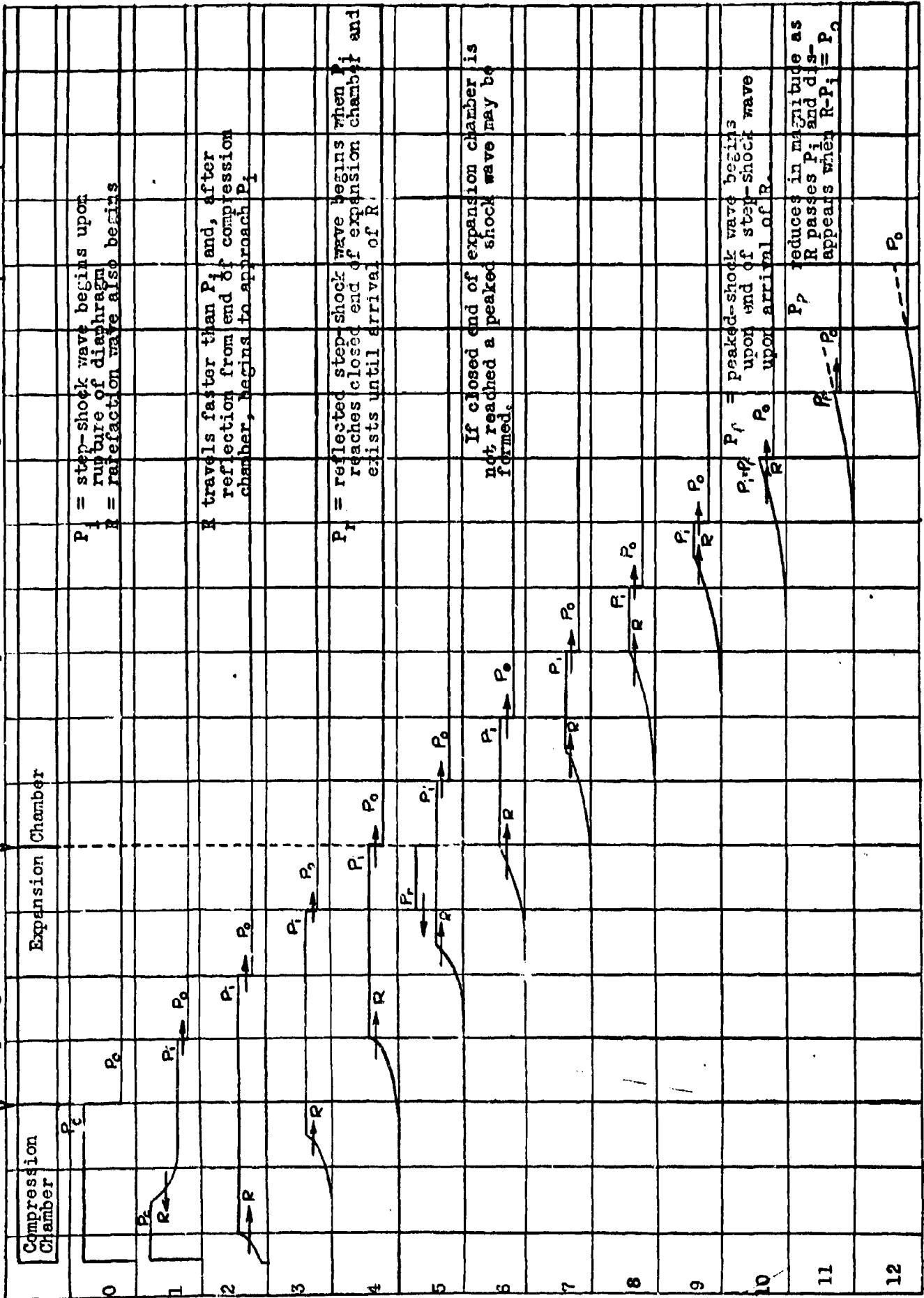
$$\frac{dE}{dx} = 5 P_0 \frac{y(y-1)}{6+y} \quad (A.7)$$

and the total energy of a shock wave of arbitrary shape is given approximately within an error of 2 percent, providing  $y$  is no larger than 3, by the following equation

$$E_t = 5 P_0 \int_0^{\bar{x}} \frac{y(y-1)}{6+y} dx \quad (A.8)$$

PRESSURE PHENOMENA IN SHOCK TUBE

Closed end of expansion chamber for production of  $P_1$ .



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